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Nash's interpretations of equilibrium: Solving the objections to Cournot

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A Nash equilibrium can also be seen as a Cournot-Nash equilibrium, though this is debated because Cournot provided a specific application, not a general formulation. In my view, another of Nash's fundamental contributions stands out when contrasting him to Cournot. Cournot treated economic decisions as optimization problems, but his stability analysis of duopoly led to endless discussions because players did not use the available information. Nash solves this with his rational interpretation: when players know the structure of the game, they can use the solution to predict the equilibrium. He thus introduces rational expectations. Nash additionally offers an adaptive interpretation: when players do not know the structure of the game, they can adjust their strategies to maximize payoffs. These adaptive expectations were anticipated by Cournot in his analysis of monopoly. In brief, Nash was not only extraordinary as a mathematician; his deep insights allow solving decades-long debates in economics.

JEL classification codes: B1, B2, B3, C7
Key words: equilibrium, rational players, consistent beliefs, adaptive expectations, rational expectations

I. Introduction

A Nash equilibrium was originally formulated by Cournot (1838) in the context of an oligopoly model, so an equilibrium point might also be seen as a Cournot-Nash equilibrium. However, this is a contested issue because the oligopoly model involves a specific application rather than a general formulation. Cournot's contribution indeed appeared in very specific attire, though the general idea behind his approach — that players select mutually optimal responses in equilibrium — had been understood before Nash (1950b), as Hurwicz (1945) for instance attests. Despite that, Cournot's contribution had a very limited impact. A key obstacle to the acceptance of the strategic analysis introduced by Cournot was the endless debate about how an equilibrium point is reached. What was found especially problematic in the stability analysis of Cournot (1838: 81-82) was that rational players did not take advantage of all the available information.[1] In my view, this leads to one of Nash's fundamental contributions.

Nash (1950c) provides an interpretation of equilibrium points that solves the decades-long debate about Cournot's dynamics with his rational interpretation: when

[*] I appreciate the suggestions of Germán Coloma, Mariana Conte Grand, Celeste González and Fernando Tohmé. My viewpoints are strictly personal.
[1] The page numbers refer to the 1897 English translation by Nathaniel Bacon.

players know the structure of the game, they can use the solution to predict the equilibrium. Though this interpretation was not published in Nash (1951), in his earlier discussion of bilateral bargaining Nash (1950a) explicitly states that the unique solution of the game provides the “rational expectations” for the players.

Nash (1950c) additionally offers an adaptive (or statistical) interpretation when the players do not know the structure of the game but they adjust their strategies to maximize their payoffs. This adaptive interpretation only became widely known later on, but this approach surfaced in game-theory texts like Luce and Raiffa (1957: 105). Evolutionary game theory developed this second interpretation (Maynard Smith and Price 1973). A specific instance had been anticipated by Cournot (1838: 53–54), when discussing how a monopolist who cannot observe the demand curve can nevertheless maximize revenues by using the elasticity of demand.

I examine the relationship between the solution concepts of Nash and Cournot in Section II, contrasting Nash’s general formulation of equilibrium points to Cournot’s specific application. In section III, I argue that the main problem with Cournot’s analysis was that his explanation of how equilibrium points are reached in a duopoly failed to persuade the economics profession. Cournot was widely, but mistakenly, interpreted as assuming that players know the model but fail to solve it correctly. In contrast, the two interpretations of equilibrium points in Nash (1950c) make what the players know about the game explicit. Section IV contains the closing remarks.

II. Equilibrium points

Nash was awarded the 1994 Nobel Prize in Economics for two fundamental contributions: distinguishing between cooperative and non-cooperative games, and providing an equilibrium concept for non-cooperative games. These contributions in Nash (1950c), his doctoral thesis, were published in Nash (1951). The solution concept was anticipated in Nash (1950b), his one-page paper on an equilibrium point. Myerson (1999: 1074) adds that this is the basic solution concept in game theory, given Nash’s insight that cooperative games where binding agreements are possible can be reduced to a larger non-cooperative game by adding the previous negotiations.

In this section I concentrate on the solution concept in Nash (1950b) and how it formalizes and generalizes the solution concept in Cournot (1838).

A. Nash's equilibrium point

Nash (1950b) takes a game with n players where each player has a finite set of pure strategies and picks a mixed strategy defined over this set. He forms a profile with the strategies of the n players. The expected payoff of each player depends on the profile of n strategies. Starting from an arbitrary profile of strategies, Nash defines what we now call a "best response profile", a profile of n strategies where the strategy of each player maximizes the payoffs in response to the strategies of the other n – 1 players in the initial profile. He defines as an equilibrium point every best response profile that is a best response to itself. As Nash (1950b) puts it,

> Any n-tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the n strategy spaces of the players. One such n-tuple counters another if the strategy of each player in the countering n-tuple yields the highest obtainable expectation for its player against the n - 1 strategies of the other players in the countered n-tuple. A self-countering n-tuple is called an equilibrium point.

After this concise and elegant definition of the equilibrium point, now known as Nash equilibrium, follows an existence proof using the Kakutani fixed-point theorem. The proofs in Nash (1950c) and Nash (1951) use the Brouwer fixed-point theorem instead.

B. Cournot's specific solution

Cournot (1838) introduces his solution concept in chapter 7 about competition among producers, after defining the demand curve $D = F(p)$ in chapter 4 and deriving the optimum price for a monopolist in chapter 5. He first analyzes the case of two producers of a homogenous good, $n = 2$. He uses the example of two springs of mineral water and initially assumes there are no production costs. Consequently, for each proprietor profit maximization reduces to revenue maximization: $\pi_1 = D_1 \cdot p$ and $\pi_2 = D_2 \cdot p$. He expresses the market price as a function of total market supply, $p = f(D)$, where $D = D_1 + D_2$.

To derive the first order conditions for an interior maximum, he differentiates these equations with respect to each proprietor's output, taking the other's output as given:

$f(D_1 + D_2) + D_1 \cdot f'(D_1 + D_2) = 0,$
$f(D_1 + D_2) + D_2 \cdot f'(D_1 + D_2) = 0.$

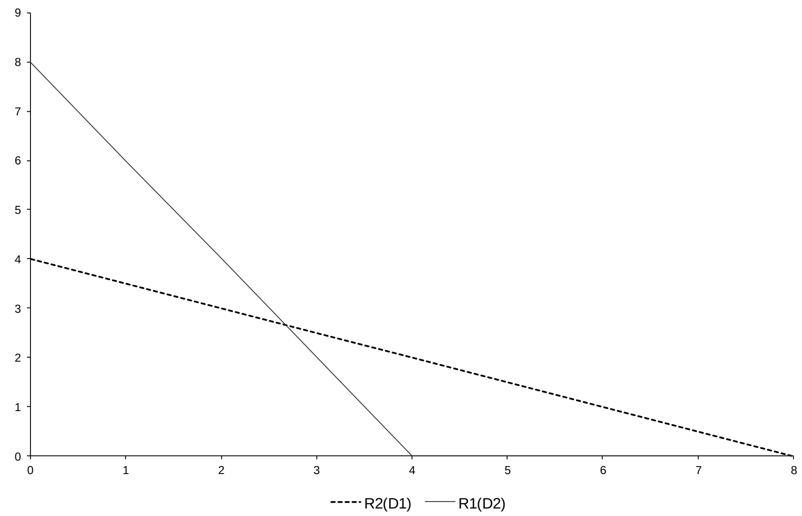
These conditions jointly determine the equilibrium point. If we assume that the demand functions take the linear form $p = a - (D_1 + D_2)$, it is simple to represent how the optimal response of each proprietor i varies in response to the other, $D_i^* = R_i(D_{-i})$, for i = 1, 2:

$D_1^* = R_1(D_2) = \frac{a-D_2}{2},$

$D_2^* = R_2(D_1) = \frac{a-D_1}{2}.$

Figure 1 depict a linear version of Cournot's reaction curves taking $a = 8$. Each curve, in what we now call "best-response function", specifies the optimal response for each level of production of the other proprietor. The intersection, which corresponds to the mutual best responses, determines the equilibrium.

Figure 1. Mutual best responses: Cournot's perspective

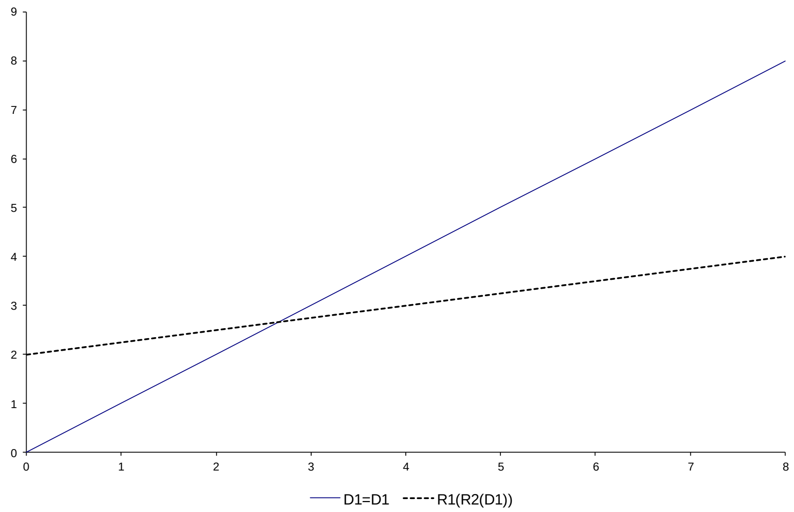


The equilibrium can also be seen as a fixed point adopting the perspective in Nash (1950c). Starting from an arbitrary point D_1, one can compute the optimal response D_2^* of proprietor 2, and the corresponding optimal response D_1^{**} of proprietor 1:

$$D_1^{**} = R_1(D_2^*) = R_1\big(R_2(D_1)\big) = \frac{1}{2}\left(a - \frac{a-D_1}{2}\right) = \frac{a}{4} + \frac{D_1}{4}.$$

This is represented in Figure 2. Optimal responses are in terms of pure strategies, since in this problem there is no mixed strategy equilibrium. In this setting, the Brouwer fixed-point theorem implies that a continuous function defined in a closed interval that takes values in that same interval must intersect the identity function. This point of intersection is the fixed point or equilibrium point $D_1^{**} = D_1$.

Figure 2. Fixed point: Nash's perspective



Cournot's solution can be seen as an instance of a Nash equilibrium. However, it is not a proper subset of the Nash (1950b) definition, since Nash considers a finite number of pure strategies while there is a continuum of strategies in this problem. What is common to both authors is the essential property that, once at equilibrium, no player will have an incentive to deviate unilaterally.

C. **Cournot as Nash's predecessor**

For Aumann (1985: 43-4), "Born more than a century ago in connection with Cournot's (1838) study of duopoly … The Nash equilibrium is the corporization of the idea that economic agents are rational, that they simultaneously act to maximize their utility. If there is any idea that can be considered the driving force of economic theory, that is it.

Thus in a sense, Nash equilibrium embodies the most important and fundamental idea of economics.” Leonard (1994: 498) considers that Aumann ignores the relevant historical issue, whether there was an actual influence of Cournot on Nash. However, from a conceptual point of view the key issue is whether the ideas are related or not.

In this same line, Martin Shubik, Nash’s fellow student at Princeton, states in a 1991 interview that he saw the resemblance from the beginning: “as soon as Nash produced the non-cooperative equilibrium, I took one look at it and said to John: this is Cournot. I don’t want to knock John’s contribution: it isn’t Cournot, Cournot is a proper subset of Nash, but it was there!” (Leonard 1994: 507). Leonard cites this testimony, together with a reference from Shubik (1989) that says that “although Cournot’s work with equilibria of games with a continuum of strategies was not strictly covered by Nash's work, conceptually Cournot’s solution could be viewed as an application of the non-cooperative equilibrium theory to oligopoly (see Mayberry et al. 1953)”. Leonard (1994: 507-9) again objects, considering that Shubik is retrospectively reinterpreting Cournot so that, in Leonard’s terms, Cournot’s dynamic idea fits in with Nash’s static idea.

A contemporaneous remark demonstrates that this is not a retrospective interpretation: Hurwicz (1953:402) literally states: “It has been pointed out by Arrow and others that the Nash solution, when it is applied to the classic oligopoly problem (the mineral water example, for instance) essentially corresponds to the so-called ‘Cournot solution’”.

As to the actual historical influence, Nash mentions Cournot in a study on bilateral negotiation published as Nash (1950a). He wrote this study for an elective course in international economics that he took as an undergraduate at the Carnegie Institute of Technology (today Carnegie Mellon University).[2] Nash states that this bargaining problem “is the classic problem of exchange and, more specifically, of bilateral monopoly as treated by Cournot, Bowley, Tintner, Fellner, and others” (Nash 1950a: 155). Kuhn (1996: 156) thinks these references were suggested by Morgenstern, because Nash had not read these authors. This seems unlikely because Cournot is not quoted once in *Theory of games and economic behavior*. The references to Cournot are

[2] At the International Workshop on Game Theory and Economic Applications of the Game Theory Society held at the Universidade de São Paulo in 2014, I heard that Nash, while still an undergraduate student, had mailed this paper to von Neumann. It seems that von Neumann threw it away without even opening the envelope. After arriving in Princeton in September 1948, Nash asked von Neumann what he thought about the paper, so von Neumann had to ask for another copy. When he read it, he immediately recommended its publication.

indirect, appearing as criticisms of the existing theories of duopoly and oligopoly. Nevertheless, I agree with Kuhn that Nash never read Cournot. Few economists have. Even among those that have read Cournot (1838), not many have paid attention to Cournot's comment in chapter 10 on international trade where he states that he will look at the case of unlimited competition (i.e., perfect competition), leaving aside the study of bilateral monopoly that is complicated and of little interest. Nash could only have gotten this reference from his professor of international economics.[3]

Hence, as to the historical record, Leonard (1994) is probably right that there was no direct influence of Cournot on Nash, foremost of all because Nash was famous for not reading almost anything. But he was also known for thinking on his feet and understanding ideas at lightning speed, so indirect influences are harder to rule out. At any rate, from the theoretical point of view the essential issue is whether it is the same solution concept or not. It is.

D. Specific application versus general formulation of a methodology

The pioneering analysis in Cournot (1838) sometimes leads the equilibrium point in non-cooperative games to be called "Cournot-Nash equilibrium", though this joint attribution is questioned.

In the first place, Myerson (1999) points out that Cournot's specific application caused a lot of confusion because it made it hard for the readers to distinguish between the general methodology and the specific model. Indeed, the lack of a general formulation in Cournot (1838) led later writers to focus on the appropriateness of Cournot's specific assumption of taking other players' output as given. For Bertrand (1883), the appropriate decision variable was price, not quantity as Cournot had proposed. Bertrand then concluded that, with two or more producers of a homogenous good, the price would fall to the competitive level.[4]

Bertrand's critique led to distinguish between Cournot's solution and Bertrand's solution of the duopoly problem (other solutions, such as Stackelberg's solution when duopolists move sequentially, were added afterwards). Today we recognize that the

[3] Cournot was then studied in international economics because he thought that Adam Smith's defense of free trade was incorrect under some circumstances (see, e.g., Fisher 1898).

[4] Within a huge literature, Png (1998) interprets Bertrand as a sealed bid auction where the lowest bidder takes all, while in Cournot's model the producers can adjust their prices in response to what other producers do. Cournot's seems to make the appropriate assumption for standard markets. Cournot (1838) adds that quantity is the decision variable as long as productive capacity is not a limiting factor.

same equilibrium concept is being applied to different models. As Myerson (1999) notes, Nash (1950b) constitutes a decisive step in understanding that.

The main objection that Myerson (1999) has for crediting Cournot (1838) with the equilibrium concept for non-cooperative games is that this would confound the application of a methodology with its general formulation. Nash (1950b) also extended the strategy space from pure to mixed strategies, and used the von Neumann-Morgenstern utility function instead of monetary payoffs. Myerson (1999) concludes that Nash's non-cooperative game theory gives the general framework to analyze, from the point of view of rational decision-makers, social problems beyond the sphere of markets. In other words, beyond the sphere of Cournot (1838).

These observations are absolutely correct, but they do not imply that Cournot cannot be given partial credit for contributing to the essential solution concept in game theory. Myerson (1999) recognizes that Cournot was aware that he was developing a general method, since he first applies the approach to *n* producers of a homogeneous good in chapters 7 and 8, and then states that he is applying the same method of reasoning to two monopolists who produce complementary inputs in chapter 9.

The generality of Cournot's approach can also be seen from another viewpoint: the strategic analysis is part of a general method of applying the calculus of extreme points to study economic decisions as optimization problems.[5] Analyzing the maxima and minima of continuously differentiable functions became the basic mathematical tool in economics for the following one hundred years, as the text on the foundations of economic analysis by Samuelson (1947: 21) attests: "The general method involved may be very simply stated. *In cases where the equilibrium values of our variables may be regarded as the solutions of an extremum* (*maximum or minimum*) *problem, it is often possible… to determine unambiguously the qualitative behavior of our solution values in respect to changes of parameters.*" Samuelson illustrates comparative statics under maximizing behavior with the example of how a production tax affects a competitive market. This is precisely the case that Cournot (1838) analyzed in chapter 8, where he also introduced the supply curve.[6] Samuelson (1947) ignores, however, the strategic analysis in Cournot's other chapters. Von Neumann and Morgenstern (1944) do not

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[5] Cournot's analysis of profit maximization was extended to utility maximization with the Marginalist revolution of Jevons, Menger, and Walras.

[6] Samuelson (1947: 22) adds that economics is not only about maximizing profits of firms or the utility of consumers, it is also concerned about the stability of equilibrium. This issue is picked up in Section III.

rescue this legacy either; unlike Samuelson (1947), they additionally reject calculus and differential equations as the main mathematical tool of economics (Leonard 1994: 494).

Perhaps the paradigmatic example of the development of Cournot's strategic approach is the Hotelling (1929) monopolistic competition model. Starting from an observation by Sraffa about the division of markets in regions, Hotelling criticizes Bertrand's idea that a competitor can capture the whole market by slightly reducing its price, since even homogeneous goods differ in other characteristics such as location. Hence, demand will vary continuously with price. To find the equilibrium, Hotelling explicitly follows Cournot's method of maximizing the profits of each monopolistic competitor by taking the other's decision as given; like Bertrand, he takes price, not quantity, as the decision variable. Hotelling then calculates optimal prices when the two locations are exogenously given (he afterwards considers what happens when locations are endogenously chosen).

The implications of the spatial model transcended economics. In a brief paragraph, Hotelling (1929: 54-55) mentions that the model can be applied to the competition between Republicans and Democrats, and can explain why parties have an incentive to be similar to each other. Downs (1957) uses the spatial model as the central analytical piece in his economic theory of democracy. In chapter 8, he shows that when voters have preferences that can be represented by single-peaked utility functions, both parties converge in equilibrium to the median voter. This is the unique equilibrium. Nash is not quoted in Downs (1957). Kenneth Arrow, Anthony Downs's Ph.D. advisor at Stanford, had been Harold Hotelling's student in Columbia (Düppe and Weintraub 2014). Down's inspiration came from somewhere else.

E. Individual rationality and strategic independence of decisions

Myerson (1999: 1072) also considers that the assumption in Cournot (1838) that the competitors make their decisions independently only became acceptable once von Neumann (1928) developed his vision about strategic independence, in what von Neumann and Morgenstern (1944) call the "normal form".

However, Myerson (1999: 1072) recognizes that von Neumann (1928) did not apply this principle consistently, since in games with more than two players he assumed that players would form coalitions. In fact, von Neumann and Morgenstern (1944: 13-15) explicitly criticize existing duopoly and oligopoly theories, and the idea that the equilibrium approaches perfect competition as the number of participants increases, on the grounds that this ignores the issue of collusion through the formation of cartels. These criticisms allude to chapter 7 of Cournot (1838), which starts with $n = 2$ and then considers an arbitrary n where each proprietor makes his decisions independently, and chapter 8 about unlimited competition (i.e., perfect competition), where price equals marginal cost. The cooperative approach developed in von Neumann and Morgenstern (1944) leads instead to expect that both proprietors will combine to exploit the market, considered as a third player.

The main snag with the Myerson critique is that Cournot (1838) did not make a mere assumption. At the beginning of chapter 7, he does assume that in the case of duopoly "the proprietors … *each of them independently* will seek to make this income as great as possible" ("des propiétaries … *chacun de son côté* cherchera à rendre ce revenu le plus grand possible"), where the italics are in the original. But Cournot (1838: 79–80) immediately contrasts the solution when decisions are made independently to the collusive solution that corresponds to the choice of a monopolist. Hence, Cournot is drawing a distinction between what are now called non-cooperative and cooperative solutions.

Furthermore, Cournot gives a strategic foundation to his non-cooperative approach. He relies on the issue of what we would today call "individual rationality", beyond what is collectively convenient: though both proprietors have a benefit from colluding to produce less and raise prices, they have a "*temporary benefit*" from deviating (Cournot 1838: 83). In other words, he points out that outside the intersection of the best-response functions at least one of the players will want to deviate. Using the normal form introduced by Borel (1921) and von Neumann (1928), this can be represented as a prisoner's dilemma. Assume each proprietor has two strategies, the collusive level and the duopoly level. The payoffs in Table 1 are the profits from each combination of production levels, and (D_2, D_2) is the equilibrium pair.

Table 1. The duopoly problem as a prisoner's dilemma

	Collusion ($D_2 = 2$)	Duopoly ($D_2 = 2.67$)
Collusion ($D_1 = 2$)	8, 8	6.7, $\underline{8.9}$
Duopoly ($D_1 = 2.67$)	$\underline{8.9}$, 6.7	$\underline{7.1}$, $\underline{7.1}$

Cournot (1838: 83) adds that an understanding between both proprietors to sustain the collusive level that a monopolist would pick requires a “formal engagement” (“lien formel”). Therefore, strategic independence is not a mere assumption. The individual incentives of each player push them in that direction. To be able to act cooperatively, cartels require a legally binding contract.

The analysis in Cournot is not obvious even after Nash (1950b, 1951). Talking about the initial reception of the Nash equilibrium, Giocoli (2003: 364-5) points out a passage where Shubik (1952:146) considers that the duopoly solution is not very relevant with complete information, because the players will strive to act cooperatively to maximize joint payoffs.[7] Perhaps the strong influence of the cooperative approach in von Neumann and Morgenstern (1944) made it hard to understand Nash himself at first, explaining why Nash’s non-cooperative approach to game theory received what Giocoli (2003) calls “a cold reception”. Shortly afterwards, Shubik himself, in Mayberry, Nash and Shubik (1953), helped to clarify the differences between cooperative and non-cooperative approaches, contrasting the non-cooperative solution in the Cournot duopoly model to several cooperative solutions that lead to collusive results.

F. Consistent beliefs: the circularity of equilibrium points

A final observation suggests that the generality of Cournot’s strategic approach had been perceived, since some economists explicitly isolated the strategic issues, distinguishing them from the specific duopoly problem.

Noteworthy is Hurwicz (1945: 909–910), who criticizes Cournot (1838) and the extant theory because there is no adequate response to the problem of what is rational economic behavior when the rationality of the actions of an individual depends on the probable behavior of other individuals. He states that Cournot tried to evade this problem by supposing the each individual knows what the others will do: “Thus, the individual’s “rational behavior” is determinate *if* the pattern of behavior of “others” can be assumed *a priori* known. But the behavior of “others” cannot be known *a priori* if the “others,” too, are to behave rationally! Thus a logical *impasse* is reached”.

[7] As Cournot (1838) realized, this requires binding agreements between the duopolists because it is not individually rational to do that. Though Shubik (1952) adds that a Nash equilibrium is impossible with incomplete information about the structure of the game, as Giocoli (2003: 364-5) comments this objection applies to Nash’s rational interpretation, not to his adaptive interpretation of equilibrium points.

This logical impasse is precisely the circularity characteristic of a Nash equilibrium: the optimal response of each player depends on what the other players are doing (Gibbons 1992: 183). Hence, it is not surprising that Hurwicz levies similar criticisms against Nash's formulation of equilibrium points. Giocoli (2003: 364) emphasizes that Hurwicz (1953) rejects the Nash equilibrium both from a prescriptive point of view of what players should do as from a descriptive one as to what people actually do. Hurwicz (1953:402) first states that, in a situation of uncertainty about what the other will do, one should follow a strategy of minimizing losses, or some other variant, even if that does not constitute an equilibrium (Luce and Raiffa 1957 explore similar arguments). Hurwicz (1953: 403) then states that not only would it be unwise to play the strategy prescribed by a Nash equilibrium, because it might lead to considerable losses if the other player does something else, but also that this is also an inadequate model to describe what people actually do.

Hurwicz (1953:403) rescues the use of the Nash equilibrium for one important class of cases, that of a competitive economy, where he considers that it is valid to take as given what the other agents do. Perhaps this can help explain why the first important impact that Nash had was due to his use of the fixed point theorem to demonstrate the existence of an equilibrium point. In their articles published in 1954, this idea of a fixed point was picked up by Arrow and Debreu as well as by McKenzie to demonstrate the existence of general equilibrium in a competitive economy (Düppe and Weintraub 2014). Of course, competitive equilibria are Nash equilibria. Cournot (1838) had established that for the competitive equilibrium of a single market.

Later game theorists ended up accepting a less stringent position, namely, that in equilibrium beliefs dictate strategies and vice-versa. This "consistent beliefs" view of equilibrium points can be derived from the formulation in Nash (1950b), where equilibrium expectations are implicitly determined by equilibrium strategies, and equilibrium strategies are a response to these implicit equilibrium expectations.

Nevertheless, Hurwicz (1945, 1953) is raising an essential issue in game theory, that equilibria depend on (i) the rationality of the players and (ii) their specific beliefs. Even among those that grasped in Cournot (1838) the general idea of an equilibrium as mutual best responses, most did not find his explanation of how these specific beliefs were formed convincing. We turn to the two interpretations of equilibrium points in Nash (1950c) which offer answers to this important critique.

III. Determining beliefs

The main obstacle to the spread of Cournot's strategic approach was not the controversy about his specific application, because other industrial organization models were developed in response. Rather, the main obstacle was Cournot's justification of how an equilibrium point is reached, which did not enjoy consensus in economics. Nash (1950c) not only provides a general formulation for equilibrium points, he also provides a rationale for beliefs that finally makes the strategic approach compelling.

A. The rejection of Cournot's dynamics

Before Nash, the main problem with the analysis in Cournot (1838) was that a convincing explanation of how the equilibrium is reached was missing. Fisher (1898), for instance, criticizes the assumption of taking the other player's strategy as given, pointing out that the dynamic process is much more complex than Cournot recognizes. Fisher then refers to Edgeworth's 1897 articles in the *Giornale degli Economisti* on the topic of how each rival forecasts the other's moves.[8] Related objections resurface in Fellner (1949), which I discuss in connection with Leonard (1994).

Leonard (1994) highlights very nicely how Cournot's strategic approach was controversial in economics mainly because of his out-of-equilibrium analysis, but he fails to draw a sharp distinction between equilibrium points and stability analysis. For Leonard (1994: 509), to talk of a Cournot-Nash equilibrium implies "a move away from viewing Cournot's analysis as dynamic, contradictory and unrealistic, to seeing it as static, coherent and useful, and thus providing a historical foundation for the Nash equilibrium".

Leonard (1994) basically leans on the criticism in Fellner (1949), for whom the behavior of the duopolists on their reaction curves was implausibly myopic: to take the other's output as given, without anticipating that the other one would react to its output decisions, is not rational outside the equilibrium point (Leonard 1994: 504-505). Since the behavior away from equilibrium was implausibly myopic, the equilibrium was nonsensical for Fellner, a point of view echoed by Leonard (1994:509).

[8] Hotelling (1929) does not object Cournot's dynamics, which may help explain why he extends Cournot's strategic approach from oligopoly to monopolistic competition.

I disagree with the indictment in Fellner (1949). This confounds two different questions. One question is: which is the equilibrium point? Another, much more difficult, question is: how is the equilibrium point reached? We still do not have a full answer to this second question. From a logical point of view, an incorrect answer to the second question does not invalidate the answer to the first. That would be like flunking a student because one of the exam questions is wrong, regardless of the answers to the rest of the questions.

In relation to how an equilibrium point is reached, Cournot (1838) was not alone in risking an answer. In his doctoral thesis, Nash (1950c: 21-24) gives two revolutionary explanations of how to motivate and interpret the equilibrium point. A way of gauging the importance of these explanations is that the long-standing objections to the explanation offered by Cournot (1838) about how an equilibrium point is reached in a duopoly had not been solved until Nash arrived. Neither interpretation appeared in Nash (1951), the published version of his thesis. This was probably a decision imposed by the editors, who typically want to save on space.

Nash offered a rational interpretation, when players know the structure of the game, and an adaptive interpretation, when they do not, that eventually made it possible to solve these objections. These interpretations can be seen as a fundamental contribution of Nash in terms of the substantive relevance of equilibria.

B. Rational expectations: Nash's "rationalistic and idealizing interpretation"

The "rationalistic and idealizing interpretation" of the equilibrium point, the second interpretation in Nash (1950c: 23), is wholly original: "a rational prediction should be unique, … the players must be able to deduce and make use of it, and … such knowledge on the part of each player of what to expect the others to do should not lead him to act out of conformity with the prediction." He adds a key precision about the player's information: "In this interpretation we need to assume that the players know the full structure of the game in order to be able to deduce the prediction for themselves."

Since the players use the structure of the game to make the prediction, Nash (1950c) is in fact introducing rational expectations.[9] Though this interpretation was not published, a similar argument appeared in his first published article which deals with

[9] Muth (1961) is the standard reference for rational expectations. He applies the idea to predict the equilibrium of a competitive market.

bilateral bargaining, where he states that his assumptions in this cooperative game lead to a unique solution that determines the "*rational* expectations of gain by the two bargainers" (Nash 1950a: 158).

Though Kuhn (2002: 48) considers that this rational interpretation could have been argued by Cournot, Cournot (1838) did not actually provide it.[10] The rational interpretation does not contradict the ideas in von Neumann and Morgenstern (1944) either. For instance, Aumann (1988: xi) traces back to von Neumann and Morgenstern (1944) the rationale "that any normative theory that advises players how to play games must pick an equilibrium in each game", when the theory recommends a unique strategy for each player. Luce and Raiffa (1957: 173) also rescue in their classic game theory book "the property that knowledge of the theory does not lead one to make a choice different from that dictated by the theory", which is "one very strong argument for equilibrium points" (Leonard 1994: 503 points out this passage). However, the emphasis is on the fact that the theoretic recommendation must respect the rationality of the players (i.e., it must be an equilibrium), not on the fact that what the players know about the game allows them to predict an equilibrium. Isolating this informational issue is, I believe, Nash's key contribution.

Luce and Raiffa (1957: 104) consider that the Nash equilibrium is not adequate for a unified theory of non-cooperative non-zero-sum games, in opposition to the current perception that it is the basic solution concept to solve games (see Ryan 2002: 128-129). For Ryan (2002: 129), the explanation is that many non-zero-sum games have multiple equilibria where the strategies are not interchangeable, so there is no clear way of how to recommend how to play the game. Luce and Raiffa (1957) thus reject the Nash equilibrium as a normative criterion of how to play a game because it does not give unambiguous advice.

Though one of the strongest initial objections to the concept of Nash equilibrium was that it might not be a unique, so it was not clear what a rational player should do, Nash (1950c) already suggests that when there are multiple equilibria heuristic reasons can lead to single out one of them. This anticipates the nice idea of equilibrium selection introduced by Schelling (1960), where the players themselves coordinate tacitly to select among Nash equilibria based on what they perceive to be focal points. The problem of multiplicity of equilibria led Selten (1975) in turn to propose another nice

[10] On the other hand, Kuhn (2002: 48) deems that Nash's adaptive interpretation is wholly original. Below, I show that this idea was anticipated by Cournot (1838) in a passage overlooked by the literature.

idea, that strategies should also be a Nash equilibrium when restricted to any subgame, which started the literature on equilibrium refinements that impose additional requirements on equilibrium points. An examination of these issues is still underway.

C. Adaptive behavior: Nash's "mass-action interpretation"

The other interpretation that Nash (1950c) offers is his "mass-action interpretation" in repeated games, where the equilibrium point arises from the action of participants that do not know the structure of the game and do not have the ability or inclination to undergo a complex process of reasoning. Instead, they accumulate empirical information about the relative advantages of each strategy. He adds that this interpretation can hold even with a small number of participants, as long as they are not aware of their interdependence.

This interpretation — his first interpretation of an equilibrium point — is formalized by Weibull (1996). This idea captures the essence of the adjustment dynamics of boundedly rational agents of evolutionary game theory. Replicator dynamics are studied in biology since Maynard Smith and Price (1973) proposed to determine which strategies are evolutionary stable. Since evolutionary stable strategies are fixed points, they are Nash equilibria. For Nakayama (2006), the very proof of the existence of equilibrium using the Brouwer fixed-point theorem in Nash (1950c, 1951) is reminiscent of the replicator dynamics of evolutionary game theory, since the probability of the pure strategies that give a larger payoff in the mixed strategy in use increases.

This interpretation was not entirely foreign to early game theory. Though Luce and Raiffa (1957) reject the concept of Nash equilibrium on normative grounds, they accept it on descriptive grounds under certain conditions, (Ryan 2002: 129). Luce and Raiffa (1957:105) accept that Nash equilibria might be relevant as a description of behavior if the equilibrium points are reached by trial and error, leading to "a set of mores and patterns of behavior" from which no individual has an incentive to deviate unilaterally. This justification is similar to Nash's adaptive interpretation of equilibrium points. This interpretation is not affected by the issue of multiplicity of equilibria, something that can be shown using replicator dynamics which select those Nash equilibria that are evolutionary stable strategies.

D. Adaptive expectations: Cournot's two dynamics

Nash's polar interpretations permit to solve the perplexity and the criticisms generated by the adjustment dynamics proposed by Cournot (1838) in his analysis of the stability of duopoly. Missing in chapter 7 of Cournot (1838) was an explicit discussion of what players actually know. Cournot was widely interpreted as assuming that the players know the structure of the game, though he never actually states that. For many readers, the myopic behavior of the players seemed a hybrid case where the players know the model but solve it incorrectly. The natural question that arose was: why didn't rational players act in a more sophisticated way?

In the dynamics originally used by Cournot (1838) in chapter 7, the sequential procedure can be represented as follows:[11]

$$D^*_{i,t} = R_i\left(D_{-i,t-1}\right).$$

Formulated this way, Cournot is introducing adaptive expectations, where each player optimizes using last period's endogenous variables to predict what other players will do this period. Though replicator dynamics can also be seen as a case of adaptive expectations, Cournot's dynamics anticipates the best-response dynamics in evolutionary game theory (Gardner 1995: 225–229). This dynamics implies that learning is faster than with replicator dynamics because each player plays a best response to what the others did in the previous period.

By strictly separating the polar cases of rational and adaptive interpretations, Nash (1950c) offers two consistent explanations, from the point of view of the information that the players possess, of how to reach an equilibrium point. The rational interpretation proposed by Nash (1950c) permits to give one resounding answer that eventually provoked a revolution in economics: if agents are rational and they have complete information, they can solve the model and jump directly to the equilibrium point, which determines the rational prediction or expectation.

On the other hand, the adaptive interpretation in Nash (1950c), when the players do not know the structure of the game but grope to find the best (local) solution,

[11] If each proprietor reacts to the other's output in the previous period, and one of the proprietors is initially on his reaction curve (e.g., the first proprietor is initially producing at the monopoly level and the entrant chooses an optimal reaction to that), one can derive this sequential process.

corresponds to an adaptive dynamics in Cournot that the economics literature has not detected, perhaps because it appears in chapter 4 where the demand curve is introduced. Cournot (1838: 53–54) states that since the demand curve $D = F(p)$ is not directly observable, it is not possible to explicitly calculate the optimal point where income is maximized by a monopolist. Cournot adds that it is nevertheless possible to find the optimum by a process of trial and error, raising the price if demand is inelastic, and lowering it if demand is elastic, since the optimum is at the point of unitary elasticity. This can also be modeled as adaptive expectations.[12]

If the players do not know the demand curve, the logical extension in chapter 7 would have been that the duopolists do not know their best-response functions either, since this requires knowledge of the demand curve. However, a trial-and-error logic similar to chapter 4 still leads to the equilibrium, since Cournot (1838) shows that the optimum for $n \geq 2$ is also at the point where the elasticity of demand that each proprietor perceives equals one (the elasticity of market demand is instead $1/n$ in equilibrium).

V. Final words

Writing this note, I realized that Nash was not only a great mathematician. His deep economic intuitions make Nash one of the greatest economists of the 20th century.

Myerson (1999: 1073) points out that Nash takes the elements in von Neumann and Morgenstern (1947) apart and reassembles them correctly, making three crucial extensions to the concept of equilibrium that appears in the oligopoly models in Cournot (1838). First, Nash analyzes the normal form of a game where the only detail is that each player has alternative pure strategies available, instead of looking at a specific game. This avoids byzantine discussions about whether the proposed model is appropriate for the concrete issue at hand. Second, Nash extends the strategy space to mixed strategies. Third, Nash uses expected utility, instead of monetary payoffs as in Cournot (1838). This makes it possible for game theory to be the foundation of the mathematical analysis of all social sciences.

Another fundamental contribution has to be added: Nash's two interpretations of equilibrium which explicitly incorporate what the players know. This allows answering

[12] In the vicinity of the equilibrium point, if there is overshooting it might be necessary to take into account more lags, e.g., the average of the past two periods.

long-standing objections to the adaptive dynamics in the duopoly model in Cournot (1838). With the rational interpretation in Nash (1950c), where the players use the structure of the game to predict the equilibrium, it becomes possible to justify the equilibrium in a convincing way. He introduces, in summary, the idea of rational expectations. Though this part of his thesis was not published, the idea appeared, with that name, in Nash (1950a). Nash (1950c) also provides an adaptive interpretation when the players do not know the structure of the game. It was later developed independently by evolutionary game theory (Maynard Smith and Price 1973). This interpretation has precedents in Cournot (1838) when discussing a monopolist's optimizing behavior.

Some brief words about John F. Nash, Jr. I met him in 2014 at a workshop organized by the Game Theory Society. He had become a living myth. I also had a chance to talk with his wife, Alicia Lardé. Knowing all they had gone through, I was struck by the fact that she only regretted not pursuing her professional career in physics. In the fifties, my parents' generation, it was common for college women to dedicate themselves to family life. When his thinking became disorganized in 1959, he resigned his position as professor at MIT. Since Alicia moved near Princeton, Nash could return to the campus. Nassar (1994), in a note in the *New York Times* that would lead to her book *A beautiful mind*, says that without the vital support of his wife, Nash would have ended up homeless. With Nash, we lost the last founding father of game theory. Alicia was extraordinary in her own way. This is a tribute to both.

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