UNIVERSIDAD DEL CEMA
Buenos Aires
Argentina

Serie
DOCUMENTOS DE TRABAJO

Área: Economía y Finanzas

INVESTMENT AND ABANDONMENT DECISIONS IN THE PRESENCE OF IMPERFECT AGGREGATION OF INFORMATION

José Pablo Dapena

Diciembre 2007
Nro. 362

www.cema.edu.ar/publicaciones/doc_trabajo.html
UCEMA: Av. Córdoba 374, C1054AAP Buenos Aires, Argentina
ISSN 1668-4575 (impreso), ISSN 1668-4583 (en línea)
Editor: Jorge M. Streb; asistente editorial: Valeria Dowding <jae@cema.edu.ar>

INVESTMENT AND ABANDONMENT DECISIONS IN THE PRESENCE OF IMPERFECT AGGREGATION OF INFORMATION

José Pablo Dapena [*]

ABSTRACT

The traditional marshallian rule of investing when the value of the investment is greater than its installment cost is modified in the presence of irreversibility and uncertainty, giving rise to an option component. Additionally, the interaction of participants holding each one a right to invest can give rise under imperfect information to situations of deviations from the optimal timing of exercise of the investment and to "herd behavior" or informational cascades given that the agents take into account when deciding not only their private set of information but also the information released to the market by the decisions made by the other agents. In the present paper we develop a model that tries to capture these effects and dynamics by showing revision of conditional expectations of the agents, and with considerations regarding the degree of dispersion of information in the economy and the effect of the number of participants and their effect into their behavior.

JEL: G00, O16, F36.
Key words: real options, capital markets, investment, aggregation, information.

[*] The viewpoints in this paper are the author's and not necessarily those of the Universidad del CEMA.

1 Introduction

The traditional marshallian rule sets investing if the present value V generated by the investment is greater than an alternative cost I. To the purpose of determining the value practitioners usually use the method of discounting future cash flows using an appropriate risk adjusted rate. However, when we take into consideration the existence of uncertainty about the value V and irreversibility of investments[1] (Dixit and Pyndick 1994), the investment decision can be associated to a call option. Myers (1977) is the first to notice that every firm is composed of two kinds of assets, those already in place and growth options. Mc Donald and Siegel (1984, 1985 y 1986) work on their research showing analogies between financial options and investment and abandonment decisions. More recently, this literature has be enriched by the work of Trigeorgis (1988, 1997), Kulatilaka (1992, 1995[a]), Grenadier (2000) and Kulatilaka (2001) about different applications of the real options framework.
From another line of research we found literature about "informational cascades" and models of aggregation of information and social learning where some sort of "herd behavior" can arise, with agents possessing imperfect and asymmetric information and learning about the true nature of the events by comparing their partial set of information with the information arising from decisions made by others. This line of research has been treated for example by Banerjee (1992), Ellison and Fudenberg (1993), Caplin and Leahy (1994) and Gale (1996) among others. The purpose of this paper is to contribute to the analysis of the behavior of agents when taking their decisions, by introducing elements of the real options literature and linking it to a framework of imperfect and asymmetric information.
As it was mentioned, the methodology of real options uses tools from financial options analysis[2] applied investment and abandonment decisions. However, while in the world of financial options the underlying value of the asset is accurately observed from the equity market, in the world of real options the value of the underlying asset is privately calculated, and hence can vary according to the perception of different agents. In this context, an agent can estimate the underlying value based on private information and based also in her perceptions of value inferred by the decisions made by other agents. This "noise" can give rise to deviations from the optimal "timing" of investment and abandonment decisions. Even more, underdeveloped capital markets can worsen this situation given that parameters of calculation of discount rates could be imperfect[3].
The present paper is based mainly on the work of Grenadier (1999) about informational externalities and release of private information by decisions of agents, and the work of Caplin and Leahy (1994) about investment decisions and herd behavior.

2 The proposition to be developed

The process of valuation of an asset to decide whether it is optimal or not to invest is somehow private and subject to perception errors because though the methods and techniques used to value future cash flows are well known, the parameters to be included can be different according to who is performing the calculation. We can suggest that there is not such a thing as "a" value in the market of real assets, therefore the valuation process can be assimilated to a process of statistical inference regarding the calculation of one unknown parameter (denoted "value" of the asset V) with incomplete information, by using tools and techniques of valuation combined with data, and hence obtaining an estimate $V_i = \theta_i V$ of the true parameter, where θ_i reflects the private perception of investor i[4]. The fact that V_i is in itself a variable subject to private considerations let us think that investors will try to calculate a range of values made by an upper and a lower boundary where they feel hey are confident that the true value

[1] For an explanation see (2004a) Annex B of Chapter II.
[2] Black and Sholes (1973) and Merton (1973) have derived under certain assumptions a closed form for the valuation of financial options.
[3] In fact development of markets makes that prices adequately convey information helping to reduce the uncertainty regarding any "true" price.
[4] This approach to valuation adopts many concepts and intuition from statistics because to value implies to calculate the value of an unknown parameter using incomplete information.

V is in. Therefore we can say for instance that the value V of an asset for agent i is $\theta_i V_i$, fluctuating with some degree of confidence in a range that goes from V_{min} to V_{max}. It is very likely that no investor will accurately calculate the true value θ, therefore it becomes useful for them to set boundary conditions for it.
Investment decisions can be associated to options and standard models for their calculation assume that their exercise is simultaneous and uninformative, and that agents perfectly know the true value of the parameters to be used in the formula and hence can decide whether or not to invest[5]. However, there are many situations in the context of real assets where agents are in possession of imperfect or private knowledge of relevant information, meaning that the estimation of value that they are doing could be subject to differences. In this context of imperfect information investors can react by calibrating their expectations taking into account what they see other agents do. This happens in every situation where decisions under uncertainty must be taken: agents tend to see what others do, therefore decisions made by participants release private information to the market and may change expectations. The herd behavior is usually associated to such environments, where informational cascades may obtain. In the context of investment and abandonment decisions regarding real assets (real options), decisions made by participants have to take into consideration the decisions made by others, given the uncertainty regarding the true value of parameters (as opposite to financial options where parameters may be well known, in real options the parameters have to be calculated) and therefore can give rise to deviations from optimal conditions of exercising.

3 Model of Analysis

We present a representative agent holding an investment option who has to decide whether to invest or not and when. She has two sources of information, her private information recognized as partial and incomplete, and the information arising from decisions made by other agents. The problem of real options is that the underlying asset could be imperfectly calculated given that the parameters are privately estimated. As a consequence the optimal exercising of the option can be different from the situation where parameters are perfectly observed. We will present a model of n risk neutral[6] agents indexed by the variable i, with $n \geq 2$ where the number of participants is of common knowledge. Each agent holds an identical investment option which can be exercised at any time (the option is perpetual and hence of the american type and decisions are taken in continuous time). The exact payoff of the option is not completely known by all agents as it depends on the aggregation of information. The optimal exercising strategy will then be contingent not only on the value of the private information but also on the information arising from decisions made by other agents.
We denote the payoff function ate exercise as $V_t - I$ and we introduce a variable θ, whose realization affects the payoff of the real option. The value of this variable is not known by all agents, given that it comes from the aggregation of all private signals, becoming of public knowledge when all the participants have exercised their options.[7] The payoff will then be:

$$\text{Max } [\theta V_t - I, 0] \qquad [1]$$

We can intuitively associate V_t to some sort of "average" or state level of the value[8]. We model the dynamics of V_t in the usual form:

$$dV = \alpha V dt + \sigma V dz \qquad [2]$$

[5] In financial options the value of the underlying asset is perfectly known from the market, knowing whether an option is "in the money" or "out of the money" which is not the case in real options, where the value has to be subjectively estimated.
[6] The assumption or risk neutrality can be easily relaxed without altering the main results.
[7] This assumption is more appropriate for private and less developed markets, where transaction costs, frictions and "noise" may perturb the prices in the market as a vehicle of information.
[8] Could be the price of the square meter in the real estate market, or the price earning ratio in the equity market

where α is the conditional and instantaneous rate of appreciation of the value, σ is the instantaneous standard deviation, and dz follows a standard Wiener process. The variable θ is made by a constant and the sum of all the private signals in the form:

$$\theta = \mu + S_1 + S_2 + \ldots. + S_n \qquad [3]$$

where μ is the expected value of θ and S_i are the independent signals, which have zero mean and positive and finite variance $V(S_i)$, Every agent knows her own signal, the parameters of the distribution of signals but does not know the real value of others´ signals. At time t=0 the exact value of θ is unknown to every agent, and each one makes an inference of it by taking into account private information. If every agent revealed the true value of the signal, then all of them can coordinate and calculate the true value of the aggregation variable; however, that is not the case, given that in some cases agents have incentives to keep their own information private and try to guess the rest[9].
Given this setup, the agent has to make an investment decision knowing:

- her own signal,
- the knowledge of decisions made by others,
- the underlying value of the state variable.

We assume two boundaries for the private signal ($S_{min} < 0$ and $S_i = S_{max} > 0$) The difference with models already treated in literature is that each agent does not known the exact value of the rest of the signals, which are inferred from the decisions already taken and from the knowledge of the distribution of signals. The stochastic process for V defined in [2] ensures that at some time T it will become optimal for the agent to invest, releasing information to the market. The fact is that agents do not know who of them holds the highest value for the signal, therefore they are continuously revising their expectations according to the decisions made by others. The investment decision process hence turns into a trade off between the benefit of waiting to see other's decisions, and investing to grab the payoff at some subjective optimal time.

3.1 The case of perfect information

It is useful to start with the case of perfect information. If that was the case and every agent made public her private signal, all the agents will be able to calibrate the value of the aggregation variable, and simultaneously exercise their options at the moment where V_t reaches the optimal value $V^*(\theta)$. By denoting $W_i(V;\theta)$ to the value of the investment option where θ is of public knowledge, $W(V;\theta)$ must solve the following equation:

$$\tfrac{1}{2} W''(V) \sigma^2 V^2 + (r - \delta) VW'(V) - r W = 0 \qquad [4]$$

where by making use that the agent is risk neutral we get that $r = \alpha + \delta$, then:

$$\tfrac{1}{2} W''(V) \sigma^2 V^2 + \alpha V W'(V) - r W = 0 \qquad [5]$$

This equation must be solved subject to appropriate boundary conditions, which ensure that the optimal strategy of investment is chosen:

$$W(V^*) = \theta V^*(\theta) - I \qquad [6a]$$
$$W'(V^*) = \theta \qquad [6b]$$

The first condition is known as "value matching", the second one is the "smooth pasting" or "high contact" condition, both ensure that he value V* is chosen to maximize the option value[10]. The solution for the value of the option W(V) and the trigger value V* can be expressed then as:

[9] The assumption of conveying information through actions and not words is of extended use in the literature. If agents were credible, they would have incentives to provide wrong information, trying to take advantage of that.
[10] The functional form of $W(V;\theta)$ is easily obtained, see Grenadier (1999).

$$W(V;\theta) = \begin{cases} (I/(\beta-1))^{1-\beta} (\theta/\beta)^{\beta} V^{\beta} & \text{for } V < V^*(\theta) \quad [7a] \\ \theta V - I & \text{for } V \geq V^*(\theta) \quad [7b] \end{cases}$$

where

$$V^*(\theta) = \frac{\beta}{\beta - 1}\frac{I}{\theta} > I \quad [8]$$

[11]

and

$$\beta = \frac{-(\alpha - \sigma^2/2) + \sqrt{(\alpha - \sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2} > 1 \quad [9]$$

where $\alpha < r$ to ensure convergence. Equation [8] represents the value of exercise $V^*(\theta)$ consistent with perfect information, where all agents invest at a time $T^*(\theta) = \inf(t \geq 0: V(t) \geq V^*(\theta))$.

4 The case of partial information

We now consider a model where agents have partial information regarding the value of the aggregation variable. At each moment of time, the set of information held by each agent contains not only the known value of her private signal, but also the inference made about the value of other's signals given the knowledge of exercise (or lack of it) by them. They update the conditional expectation of θ_i by observing what other agents have done or not.

4 .1 Equilibrium with information revelation

We derive the equilibrium for a game of $n \geq 2$ agents. The agents know the value of their private signal and they infer the value of the aggregation variable by seeing what others do or not. At some moment of time the state variable reaches the trigger value V^*_i for agent i according to [7] and [8] and consistent with S_i, which presses the agent to make a decision between:

- waiting to see what other agents do,
- killing the option and investing.

Every agent will calculate an optimal value V^*_i according to [8] indicating the subjective trigger at which she would exercise her option based in her own estimation of θ, which happens at time T^*_i. Given the distribution of S_i between the two boundaries, this decision will have to be evaluated first by the agent holding the highest value of the signal, now denoted 1. However, given the aggregation of information and the lack of coordination, at that moment the agent will have to revise her conditional expectation of the aggregation variable *given that no one has taken a decision*.

4.2 Equilibrium with two agents

Given that the variable V follows the process set in equation [2] and given the private signals S_i, the investment strategy for every agent $i = 1,2$ will be contingent on:

---

[11] We see that V has to be well above the cost of investment I for an agent to invest, because there is a value in the waiting option under uncertainty.

- the current value of V_t
- her signal S_i
- the strategy of exercise followed by the other agent.

The payoff at the time of exercise is θ^*V_t - I. For the purpose of the model we assume that signals follow a uniform distribution[12] in the space $[S_{min}, S_{max}]$:

$$S_i \in [S_{min}, S_{max}],\ i = 1,2,\ \text{with } S_{min} < 0,\ S_{max} > 0 \qquad [10]$$

giving rise to the aggregation variable

$$\theta = \mu + S_1 + S_2 \qquad [11]$$

where

$$\mu > S_1 + S_2 \qquad [12]$$

which guarantees exercise of options at some finite time T[13].

We denote the investment decision by the binary variable[14]

$x_i = 0$ if agent i chooses not to invest [13a]

$x_i = 1$ if she chooses to invest (13b]

Each agent i has an initial expectation about the value of the signal of the other agent j conditional on not exercise of the option (x_j=0)

$$E_i\left[S_j \big/ x_j = 0 \right] = 0 \qquad [14]$$

which in turns means that the expected value of θ for agent i is

$$E_i\left[\theta \big/ x_j = 0 \right] = S_i \qquad [15]$$

The dynamics of V_t according to [2] and the fact that agent 1 has the highest value for the signal will make that at some time T^*_1 the state variable V will reach the optimal value V^*_1 for agent 1 according to [8] and conditional on what she thinks about the signal of agent 2, given that until that time the conditional expected value of θ was S_1 according to [15]. At this moment occurs the first relevant event, where agent 1 has to decide whether to invest or not: if agent 1 realizes that if she was alone in the market that would be the time T^*_1, then she revises her expectation of θ taking into consideration the fact that agent 2 has not yet invested, making her change her conditional expectation of the aggregation variable θ given that now she thinks that the signal S_2 of the other agent has to be between S_{min} and S_1 giving rise to the new conditional value θ:

$$E_1\left[\theta \big/ x_2 = 0 \right] = S_1 + \left(\frac{S_1 + S_{min}}{2} \right) < S_1 \qquad [16]$$

---

[12] The assumption of uniform distribution makes more tractable the model, but other distributional forms can be used.
[13] From here onwards we normalize θ in [11] from μ .
[14] This is a key aspect of this paper. The signal S_i is continuous in the space defined, while the decision observed adopts the form of a binary variable, being x_i an imperfect indicator of the true value of S_i and hence of θ.

and hence agent 1 has to wait until the state variable reaches a higher value given that lack of exercise by agent 2 has changed agent 1´s conditional expectation. The agent 1 does not invest given that change in expectations has increased the trigger value V^*_1 to V^{**}_1 consistent with [16].
In the same way, even in the case that the signal of agent 2 is below to that of agent 1, and if agent 1 has not yet invested, the conditional value of the aggregation variable θ for agent 2 is similar the previous case according to [15]:

$$E_2\left[\theta / x_1 = 0\right] = S_2 \quad [17]$$

where at some time T^*_2 Vt reaches V^*_2 and it becomes optimal for agent 2 to invest given the value of her private signal and considering that agent 1 has not yet invested. At that time, agent 2 thinks that she has the highest signal (she does not know that agent 1 has indeed the highest signal), and similarly to the process of revision of expectations undertaken by 1, she revises her conditional expectation of θ, where the lack of exercise of the option by 1 lets her assume her signal S_2 is highest and hence she reacts by increasing her optimal trigger value V_2^* to V^{**}_2 in the same way as investor 1:

$$E_2\left[\theta / x_1 = 0\right] = S_2 + \left(\frac{S_2 + S_{min}}{2}\right) < S_2 \quad [18]$$

where

$$E_1\left[\theta / x_2 = 0\right] > E_2\left[\theta / x_1 = 0\right] \quad [19]$$

given that the signal of 1 is higher than that of.

The dynamics of Vt leads that at some time T^{**}_1 for a trigger value V^{**}_1 according to [8] it becomes optimal for agent 1 to invest given the value of her conditional expectation of θ from [16], taking as given that agent 2 has not yet exercised her investment option.
Given that according to [19] the conditional expected value of θ is greater for agent 1 than for agent 2, it follows that the trigger value will be lower for agent 1 $V^{**}_1 < V^{**}_2$ according to [8], where the first agent that will have to decide whether to invest or not at time T^{**}_1 will be agent 1. When V_t=V^{**}_1, where θ_1 according to [16], agent 1 finds optimal to invest and release such information to the market, based on her private signal and on the fact that the other agent has no invested.
Whit respect to agent 2, and knowing that agent 1 has now invested, we can find two situations:

- agent 2 has revised her conditional expectation of the aggregation variable (conditional on the fact that agent 1 has not invested),
- agent 2 has not revised her conditional expectation given that the state variable V has not reached her trigger value.

If V_t has not yet reached the critical value V^*_2 which triggers the first revision of expectations by agent 2, e.g. from:

$$E_2[\theta] = S_2 \quad [20]$$

to

$$E_2\left[\theta / x_1 = 0\right] = S_2 + \left(\frac{S_2 + S_{min}}{2}\right) \quad [21]$$

it would mean that the state variable has not reached the critical value V^*_2 for agent 2, and therefore agent 2 has not time to revise her conditional expectation because she is surprised by the decision of agent 1 which in turn gives her partial information regarding the value of agent 1´s signal, therefore with this information in hand she revises her conditional expectation according to [20] in the form:

$$E_2\left[\theta / x_1 = 1\right] = S_2 + \left(\frac{S_2 + S_{max}}{2}\right) \qquad [22]$$

where she infers that agent 1´s signal has to be between her own signal and the upper bound.
The alternative situation (see graph 1) is the one where agent 2 has time to revise her conditional expectation of the value of the aggregation variable because the state variable reaches the critical value V^*_2 and agent 1 has not yet invested, making her think she has the highest value for the signal S_i. Agent 2 then lowers her conditional expectation about S_1, which after some period of time is revised again when she sees agent 1 decision of investing:

$$E_2\left[S_1 / x_1 = 1\right] = \frac{S_2 + S_{max}}{2} \qquad [23]$$

when she observes the decision of investing by agent 1, where the conditional expected value of θ for agent 2 will be:

$$E_2\left[\theta / x_1 = 1\right] = S_2 + \left(\frac{S_2 + S_{max}}{2}\right) \qquad [24]$$

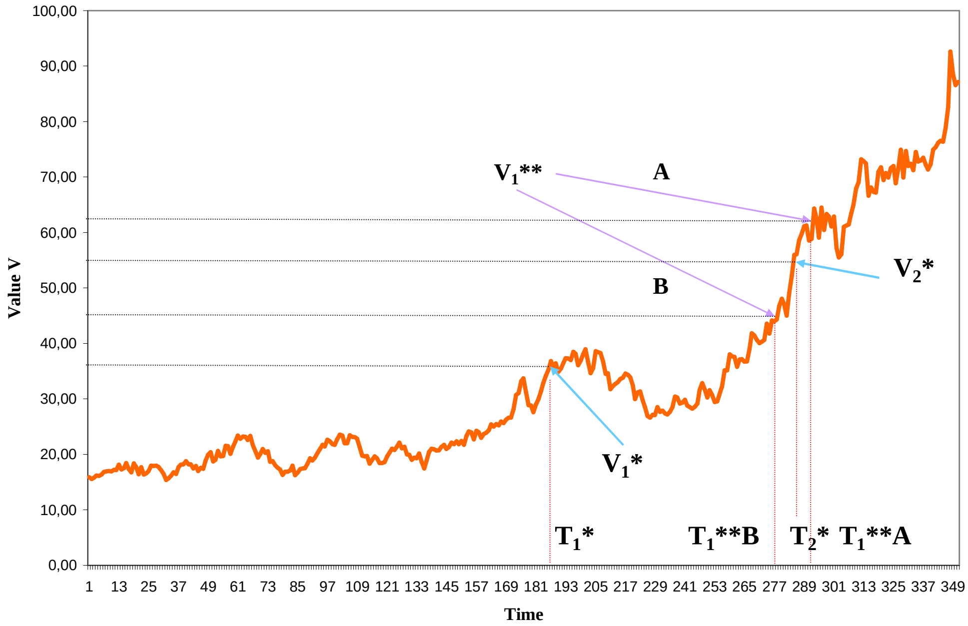
which in turn is greater to the conditional expectation of θ given the absence of investment by agent 1:

$$E_2\left[\theta / x_1 = 0\right] = S_2 + \left(\frac{S_2 + S_{min}}{2}\right) \qquad [25]$$

Agent 2 revises her optimal investment strategy according to the revelation of information released by agent 1.

The following graph allows us to better see the dynamics of decisions of the agents.

Graph 1 Simulated dynamics for state variable V and trigger values for decisions



From the graph we observe that the trigger value V^*_i is reached first by agent 1 at the critical value V^*_1 given her signal is the highest, but here she revises down her conditional expectation of the aggregation variable, and the investment decision is taken when the state variable reaches the second critical value V^{**}_1 (consistent with the revised θ according to [16]); regarding agent 2 the two situations mentioned are labeled A and B, where in A investor 2 has time to lower her expectation according to [21] so when she sees agent 1´s decision of investing she revises up again the conditional expectation of according to equation [22]; in B agent 2 has no time to lower her expectation because agent 1´s decision of investing is made before the state variable reaches the first critical value for agent 2, so she revises up her conditional expectation of the aggregation variable according to [22] without passing through the reformulation of equation [21]. Before continuing, it becomes relevant to state the conditions under which: -agent 2 has time to first revise down her conditional expectation $E_2[\theta]$ and the revise it up when agent 1 invests, -or she is surprised by the decision of agent 1 without passing through the first revision, because at time $t<T^*_2$ she observes agent 1 investing:

For agent 2 to revise down her expectation means to change at time T^*_2 her estimation of θ according to [20] and [21] given the absence of decision by agent 1. Taking into account the information held by agent 1, she decides to invest when the value Vt reaches the critical value V^{**}_1 consistent with the new estimation of θ for agent 1 according to [16].

Agent 2 <u>will not have time</u> to revise down her conditional expectation if agent 1 invests before the state variable reaches the critical value V^*_2 for agent 2, which will happen when:

$$S_2 < S_1 + \left(\frac{S_1 + S_{min}}{2}\right) \qquad [26]$$

which leads agent 2 to revise her conditional expectation of the aggregation variable according to equations [20] and [24]. If this was not the case, then agent 2 has time lo revise down her conditional expectation, and the revise it up when she sees agent 1 investing.

There are two points that deserve special treatment in the model. The first point is regarding the timing of the investment, in the sense of evaluating the conditions under which agent 1 will decide to invest before the optimal time T* with perfect aggregation of information, and when she will invest after that time (excessive waiting). The second point that deserves special attention is the conditions under which we can find an informational cascade where the decision made by agent 1 automatically triggers investment by agent 2. Now we treat the two points separately.

4.2.1 Timing of investment

If there were no information aggregation problems and signal were of public knowledge (e.g. $\theta = S_1 + S_2$ is perfectly known) then there would be an optimal timing of investment T* consistent with θ where everybody invests at the same time and coordinated, according to equations [7] and [8]. The private feature of signals makes the agent with the highest signal to first enter the market, which in turn has not to coincide with the optimal time T*. As it was already shown, when the value of the state variable V_t reaches the critical value V^*_1 in T^*_1, agent 1 realizes she may have the highest signal; the fact that the other agent has not invested yet makes agent 1 revise her expectation of S_2 from $E_1[S_2] = 0$ to $E_1[S_2/x_2=0] = (S_1 + S_{min})/2$, where $E_1[S_2] > E_1[S_2/x_2=0]$ in line with the mechanism already described on the previous section. This situation is not trivial: agent 1 exercises her investment option at T^{**}_1, later than time T^*_1 where she would have invested had not taken into account information regarding the expectation of agent 2´s signal[15]. Agent 1 chooses to incorporate information coming from the market and decides to wait instead of deciding along with her own signal S_1 only. As it was mentioned, agent 2 knows the true value of her signal S_2, while agent 1 estimates it conditional on the observation of binary variable of investing or not and on the boundaries set for S. Agent 1 would be investing at the optimal time $T^* = T^{**}_1$ if and only if her conditional expectation of the signal of agent 2 coincides with the true value,

$$E_1\left[{}^{S_2}\!\big/_{x_2 = 0}\right] = S_2 \qquad [27]$$

where it follows that

$$\frac{S_1 + S_{min}}{2} = S_2 \qquad [28]$$

and

$$S_1 = 2S_2 - S_{min} \qquad [29]$$

Every time the left side of [29] is greater than right side, agent 1 will be overestimating the true value of the signal of agent 2, making that the decision of investing is made at a time T previous to the optimal, $T^{**}_1 < T^*$, where agent 1 hurries to invest. If on the contrary, the left side of [29] is smaller than the second, agent 1 will be underestimating the true value of the signal of agent 2 S_2, where agent 1 will enter after the optimal time, $T^{**}_1 > T^*$.

4.2.2 Consecutive decisions of investing

We have derived the time T^{**}_1 at which agent 1 decides to exercise her option, now we will treat the conditions under which agent 2 decides to immediately invest after seeing agent 1´s decision. Likewise agent 1, agent 2 does not know whether her signal is the highest and her conditional expectation of agent 1´s signal is $E_2[S_1] = 0$ according to (20). Given the dynamics of V_t, there will be a time T^*_2 where it becomes optimal for agent 2 to invest taking into account that agent 1 has not invested. This time T^*_2 can be greater or smaller that T^{**}_1: if $T^*_2 < T^{**}_1$, agent 2 has time to revise down her conditional expectation of agent 1´s signal S_1 according to the conditions derived in [26]). Given that agent 1 has not yet invested, agent 2 revises her conditional expectation of agent 1´s signal S_1 moving

[15] States the difference between investing by using only private information or including information released to the market by others.

the expectation from $E_2[S_1] = 0$ to $E_2[S_1/x_1=0] = (S_2 + S_{min})/2$, where $E_2[S_1] > E_2[S_1/x_1=0]$ according to [20] and [21]. For this situation to happen, it must follow that

$$S_2 > S_1 + \left(\frac{S_1 + S_{min}}{2}\right) \qquad [30]$$

meaning that the signals have to be <u>close to each other</u>.

If that was not the case, then agent 2 has no time to revise down the conditional expectation about agent 1´s signal and hence her estimation of the aggregation variable θ, making to revise it up (alternative A in graph 1). Given this, we now formulate the condition that has to be satisfied for agent 2 to immediately follow agent 1´s decision. Agent 2 will immediately exercise her option if the expected conditional value of the aggregation variable θ for agent 2 given that agent 1 invests according to [25] is greater than the expected conditional value of θ for agent 1 according to [16]:

$$E_2\left[\theta / x_1 = 0\right] = S_2 + \frac{S_2 + S_{min}}{2} > E_1\left[\theta / x_2 = 0\right] = S_1 + \left(\frac{S_1 + S_{min}}{2}\right) < S_1 \qquad [31]$$

or alternatively:

$$S_2 > S_1 - \frac{1}{3}\left[S_{max} - S_{min}\right] \qquad [32]$$

The last equation shows that for agent 2 to immediately follow agent 1, the signals should be sufficiently close to each other, regardless the fact that private estimations of the trigger value of the state variable are differently for the two agents; in fact, the two agents invest with different estimations of the value $\theta\ V_t$ ($\theta_i\ V_t$ in each case).

The critical value V_t diminishes for agent 2, from V^*_2 to V^{**}_2, given that the new conditional value of θ is greater than the one previous to the expectation adjustment based on the investing decision of agent 1. This situation motivates that agent 2 eventually decides to invest immediately, following agent 1´s decision. The effects arising are:

- we can find investment made either before or after the optimal time as a consequence of lack of coordination between participants. This effect is different form the one shown on Grenadier (1999).
- We can find an informational cascade causing that agent 2 immediately follows agent 1´s decision after revising her conditional expectation.
- a point that differs from Grenadier (1999) is that no agent knows who of them has the highest signal until information is revealed to the market.

4.3 The case of n agents or investors

In this section we extend the model to treat the case where we have n > 2 agents holding each an investment option. Each agent holds an investment option whose payoff at exercise is $\theta\ V_t$ - I. We again assume a uniform distribution for the private signals in the space:

$$S_i \in [S_{min}, S_{max}],\ i = 1,\dots, n, \text{ with } S_{min} < 0,\ S_{max} > 0 \qquad [33]$$

where agents are indexed from 1 to n based on the value of their private signals, $S_1 > S_2 > S_3 > \dots S_n$, giving rise to the aggregation variable:

$\theta = \mu + S_1 + S_2 + \ldots + S_n$ [34]

where again

$\mu > \Sigma (S_i)$[16], for i= 1…n [35]

which guarantees exercise of options in finite time like [12]. We again denote the decision of investing or not by a binary variable according to [13a] and [13b] for each investor i.
Agents only know in the beginning the value of their own signal, giving rise to the expectation of the value of signals of the rest of agents j ($j \neq i$) conditional on $x_j=0$

$$E_i\left[S_j / x_j = 0 \right] = 0 \forall \ldots j \neq i \quad [36]$$

which in turn means that the expected value of θ for agent i is:

$$E_i\left[\theta / x_j = 0 \right] = S_i \quad [37]$$

The agent with the highest level for her private signal is labeled 1, and henceforth. The dynamics of V_t according to [2] will make that at some time $T^*_1 < \infty$ V_t will reach the initial critical value V^*_1 for agent 1 according to [8], knowing that until that time no one has exercised her option, where $E_1[\theta] = S_1$ according to [37]. In the same tense as in the case of n=2 agents, agent 1 revises her expectation regarding the value of the aggregation variable θ by estimating that the rest of the signals are situated between S_{min} y S_1 for each agent j ($j\neq 1$), giving rise to the new conditional expected value for the aggregation variable for agent 1:

$$E_1\left[\theta / x_j = 0, \forall j \neq 1 \right] = S_1 + (n-1) * \left(\frac{S_1 + S_{min}}{2} \right) < S_1 \quad [38]$$

Under this revision, agent 1 has to wait more until V_t reaches a higher second critical value V^{**}_1 consistent with the process of revision of expectations by agent 1 in the form:

$$E_i\left[\theta / x_j = 0, \forall j \neq i \right] = S_i + (n-1) * \left(\frac{S_i + S_{min}}{2} \right) < S_i = E_i[\theta] \quad [39]$$

which according to [8] implies a $V^{**}_1 > V^*_1$ in the same way as when there were two participants, but with the difference now that the increase in the number of participants has amplified the conditional estimation of the other agents´ signals which in turn increases more the critical value V^{**}_1 in comparison with the case of n=2 agents. Each agent j ($j\neq 1$) performs a similar process of revision of expectations regarding the value of θ

$$E_j\left[\theta / x_i = 0 \right] = S_j \text{ for all } i \neq j \quad [40]$$

where there will be an optimal timing T^*_j for each agent $j\neq i$ to invest, unless those with the highest signals find optimal to invest before the revision of the expectations by those with lower signals. For each agent j we find two alternative situations:

[16] It is valid the simplification made in footnote 14.

- agent j revises down her conditional expectation of the aggregation variable θ given no one has invested yet.
- agent j observes that some one invests before she has time to revise down her expectation of θ, and hence she revises it up given the information released to the market.

Conditions for each situation to happen have been of treatment in the previous section, so it does not become relevant to repeat them now. It is worth however to mention that any of the two situations can happen: agent 2 (and the rest) revises her expectation taking into consideration the presence or absence of investments. In this last case agent 2, who does not know that agent 1 holds the highest signal, revises up her conditional expectation of θ in the some way done by agent 1, giving rise to the new critical value $V^{**}_2 > V^{*}_2$ arising from:

$$E_2\left[\theta / x_i = 0, \forall i \neq 2\right] = S_2 + (n-1) * \left(\frac{S_2 + S_{min}}{2}\right) \qquad [41]$$

where

$$E_1\left[\theta / x_i = 0, \forall i \neq 1\right] > E_2\left[\theta / x_i = 0, \forall i \neq 2\right] \qquad [42]$$

The rest of the agents (3 to n) act in a similar way.

The dynamics of V_t leads to some time T^{**}_1 with associated value $V=V^{**}_1$ where it becomes optimal for agent 1 to invest based on her conditional revised expectation of the aggregation variable θ considering absence of investments by the other participants according to [39]. Agent 1 releases information to the market with her decision meaning that agent 2 (and the rest of agents) has to revise her conditional expectation in the form:

$$E_2\left[S_1 / x_1 = 1, x_i = 0, \forall i \neq 1,2\right] = \frac{S_2 + S_{max}}{2} \qquad [43]$$

where the expected conditional value of θ for agent 2 will be:

$$E_2\left(\theta / x_1 = 1, x_i = 0, \forall i \neq 1,2\right) = S_2 + \left(\frac{S_2 + S_{max}}{2}\right) + (n-2) * \left(\frac{S_2 + S_{min}}{2}\right) \qquad (44)$$

The last equation shows that agent 2 takes into consideration the act of investing made by agent 1, and the absence of investments by the other agents.
Every agent i (apart from 1 and 2) undergoes a similar process of revision of expectations, giving rise to a new set of revised expectations for each one. For example, if agent 3 sees that agent 1 invests and the others do not, she revises her expectation of θ in the form

$$E_3\left[\theta / x_1 = 1, x_i = 0, \forall i \neq 1,3\right] = S_3 + \left(\frac{S_3 + S_{max}}{2}\right) + (n-2) * \left(\frac{S_3 + S_{min}}{2}\right) \qquad [45]$$

while if she observes that both agent 1 and 2 invest, she revises her expectation in the following way:

$$E_3\left[\theta / x_{1,2} = 1, x_i = 0, \forall i \neq 1,2,3\right] = S_3 + 2 * \left(\frac{S_3 + S_{max}}{2}\right) + (n-3) * \left(\frac{S_3 + S_{min}}{2}\right) \qquad [46]$$

The rest of agents follow a similar pattern of revision of expectations according to their own signals and the decisions made by other agents.
As in the case of two agents, there are two situations worth analyzing:

- the situation where the time of the investment decision made by agent 1 differs from the optimal under perfect aggregation of information;
- the situation where agents behave like a herd when they see decisions made by other agents

4.3.1 Timing of the investment
The first situation regards to the conditions under which the agent with the highest signal (indexed as 1) invests at a time different from the optimal with perfect aggregation. In the same tense as in the case of n=2 agents described in section 4.2.1, the decision made before or after the optimal timing under perfect aggregation will depend upon expectations formed by agent 1. She initially has an expectation of θ equal to S_1, which is the revised when it becomes optimal to invest under this situation and she sees no one has yet invested (time T^*_1). The revision of expectations proceeds in the following result:

$$E_1\left[\theta / x_i = 0, \forall i \neq 1\right] = S_1 + (n-1) * \left(\frac{S_1 + S_{min}}{2}\right) \quad [47]$$

when now V_t reaches the new critical value V^{**}_1 according to equation [8] consistent with the revised conditional expected value of θ according to [47], agent 1 decides to exercise her option and becomes the first to do it. Under perfect coordination, all agents should exercise their options simultaneously when the state variable V_t reaches a trigger or critical value V* according to [8] consistent with $\theta = \Sigma S_i$[17], i=1…n; this means that agent 1 will invest before the optimal case if:

$$S_1 + (n-1) * \frac{S_1 + S_{min}}{2} > S_1 + \sum_{i \neq 1}^{n} S_i \quad [48]$$

and rearranging:

$$S_1 > 2 * \frac{\sum_{i \neq 1}^{n} S_i}{n-1} - S_{min} \quad [49]$$

where

$$S_i > 2 * \overline{S_{i \neq 1}} - S_{min} \quad [50]$$

which is a very similar condition to that obtained for the case of n=2 agents in [28] and [29], where S_2 has been replaced by the average value of the signals of the agents without including agent 1.
If the inequality follows the direction stated in [50], agent 1 would be overestimating the true value of the average value of signals of the rest pf the agents at the time of her decision, and hence investing before the optimal time. In the same way, we will find excessive waiting if

$$S_1 < 2 * \overline{S_{i \neq 1}} - S_{min} \quad [51]$$

because agent 1 underestimates the average value of the signals of the rest of participants.
In both cases the "leading investor" (named i=1) deviates from the optimal time T* under perfect coordination.

[17] $\theta = \mu + \Sigma S_i$ if we show the original condition.

4.3.2 Consecutive investment decisions

The second point to be analyzed regards the behavior of agents and the sequence of investment decisions made by them. As we have shown, there is a time T^{**}_1 at which agent 1 finds optimal to invest given the value of the revised conditional expectation of the aggregation variable. The decision releases information to the market and reveals it to the other agents, who can now revise their own conditional expectation of such variable (remember investors are indexed from 1 to n according to the level of their private signal in the form $S_1 > S_2 > S_3 > \ldots > S_n$). The agent labeled 2 has to decide whether to invest or to wait at the light of the new information arising from agent 1´s decision. For agent 2 to follow agent 1´s decision and invest, it must happen that her revised conditional expectation according to the following equation:

$$E_2\left[\theta \big/ x_1 = 1, x_i = 0, \forall i \neq 1,2\right] = S_2 + \left(\frac{S_2 + S_{max}}{2}\right) + (n-2) * \left(\frac{S_2 + S_{min}}{2}\right) \qquad [52]$$

is greater than the expected conditional value of the aggregation variable θ for agent 1,

$$E_1\left[\theta \big/ x_i = 0, \forall i \neq 1\right] = S_1 + (n-1) * \left(\frac{S_1 + S_{min}}{2}\right) \qquad [53]$$

or rearranging terms:

$$S_2 < S_1 - \frac{1}{n+1} * \left[S_{max} - S_{min}\right] \qquad [54]$$

which is the extension of condition [32] for the case of n participants. Agent 2 will follow agent 1´s decision if their signal are close enough (however not equal), with distance no larger than a critical distance given by

$$d = \left[\frac{S_{max} - S_{min}}{n+1}\right] \qquad [55]$$

Now we consider the case of agent 3, where two situations can be analyzed:

- the situation where agent 2 has decided not to invest, because her signal is at more distance than the critical distance d
- the situation where agent 2 has decided to invest.

If the first situation was the case, then agent 3 sees that only one agent has decided to invest, so she does not invest because her signal is lower than agent 2´s, who has decided not to invest.
However, if agent 2 has decided to invest, now agent 3 sees two agents investing, so her revised conditional expectation must reflect this situation. According to equations [52], [53], and [54], agent 3 will invest and follow agent 2´s decision if when revising her conditional expectation:

$$E_3\left[\theta \big/ x_{1,2} = 1, x_i = 0, \forall i \neq 1,2,3\right] = S_3 + 2 * \left(\frac{S_3 + S_{max}}{2}\right) + (n-3) * \left(\frac{S_3 + S_{min}}{2}\right) \qquad [56]$$

she gets that is greater than the conditional expected value of the aggregation variable θ for agent 1 as shown in equation [53], where by rearranging terms allows us to show the critical condition:

$$S_3 < S_1 - \frac{2}{n+1} * \left[S_{max} - S_{min}\right] \qquad [57]$$

The last equation shows that agent 3 will invest and follow agent 1 and agent 2 of her signal is sufficiently close to the one held by agent 1 (e.g., the distance between both is not greater than the critical distance d expressed in [55]):

$$S_3 < S_1 - 2 * d \qquad [58]$$

As a matter of fact, agent 2 has decided to invest, so her signal is at no greater distance of agent 1´s signal than the critical distance d according to [54] and [55], so it is a sufficient condition for agent 3 to immediately invest that her signal is at no greater distance of agent 2´s signal than:

$$S_3 < S_2 - \frac{1}{n+1} * [S_{max} - S_{min}] \qquad [59]$$

In a similar way and more generally, agent indexed as j will immediately exercise her option given that the previous agents have decided so if

$$E_j\left[\theta / x_j = 1, \forall i < j, x_i = 0, \forall i > j\right] = S_j + (j-1) * \left(\frac{S_j + S_{max}}{2}\right) + (n-j) * \left(\frac{S_j + S_{min}}{2}\right) \qquad [60]$$

is greater than the conditional expected value θ for agent i,

$$E_i\left[\theta / x_i = 0, \forall i \neq 1\right] = S_i + (n-1) * \left(\frac{S_i + S_{min}}{2}\right) \qquad [61]$$

or rearranging:

$$S_j < S_i - \frac{(j-1)}{n+1} * [S_{max} - S_{min}] \qquad [62]$$

which is similar to the conditions derived in [57] and [58] where immediate exercise of investment options happen for agents with signals sufficiently close to each other, distance between them not greater than the critical distance d:

$$S_j - S_{j-1} < d \qquad [63]$$

given that all agents with signals greater than agent j´s have chosen to exercise and reveal information.
In case that inequality shown in equation [63] was verified in the opposite direction, then agent j does not copy the investment decision made by agent j-1 and waits until the state variable V_t reaches the critical value V^{**}_j.
We can see from the calculations that the distance between signals has to be sufficiently wide (greater than critical distance d) to stop the chain of immediate exercise of investment options. Even more, once the chain is cut where agent z decides not to invest and wait until the state variable V_t reaches the trigger value V^{**}_z in line with her revised conditional expectation, the rest of agents will not invest either, waiting as well.
Immediate investment decisions will take place for all agents with signals sufficiently close (e.g. distance smaller than critical distance d according to [55]), even in the case that their signals are not the same. The element giving rise to this situation is the fact that decisions are shown as a binary variable, while signals are in continuous time. This situation can give rise to more volatile investment behavior, generated by the new arrival of information regarding decisions.

4.4 Sensitivity of the behavior of agents to a change in parameters.
In this section we develop about the sensitivity of the variable of critical distance d to changes in relevant parameters:

- the degree of dispersion of signals among participants (measured by the range S_{max} - S_{min})
- the number n of participants holding options.

Recreating [55],

$$d = \left[\frac{S_{max} - S_{min}}{n+1} \right] \qquad [55bis]$$

we observe that an increase in the range S_{max} - S_{min} positively affects the critical distance d and hence the likelihood of immediate decisions of investment (sort of herd behavior) in the form:

$$\frac{\partial d}{\partial [S_{max} - S_{min}]} = \frac{1}{1+n} > 0 \qquad [64]$$

given S_{max} - $S_{min} > 0$ which is the case. The intuition arises from the fact that an increase in the range of possible values for the signals makes more difficult for agents to estimate the true value of the signals of the rest of the participants (on the extreme, if there was no range, estimation would take place without error and perfect coordination will obtain).

The other relevant aspect is to analyze the impact that the number of participants have in the critical distance d and hence in the likelihood of immediate sequence of investment decisions. The value of the partial derivative is negative:

$$\frac{\partial d}{\partial n} = [S_{max} - S_{min}] * \left(\frac{-1}{(1+n)^2} \right) < 0 \qquad [65]$$

which says that an increase in the number of participants reduces the value of the critical distance d and therefore makes more unlikely to obtain immediate investment decisions.

In brief, from the comparative static we see that the less the number of participants in the market, and the more the dispersion of private signals, the more likely becomes to obtain immediate investment decisions. If we take into consideration that the role of markets is to convey information to a great number of agents, then we can suggest that underdeveloped capital markets (where investment takes place) may give rise to such problems like the ones shown here.

5 Model for abandonment decisions

We can model a similar framework like the one shown before for agents holding abandonment options; written on an asset whose value follows equation [2]´s dynamic and with aggregation variable as shown in [3].
In this case, the agent holds a perpetual option to abandon an asset, getting in exchange a liquidation value labeled L[18], from where the payoff function will become:

[18] Which can be associated to a recovery value of the asset. In corporate finance can be associated to the face value of debt when it is used as a source of financing, where V is the market value of the collaterals.

$$\text{Max}\,[L - \theta V_t, 0] \quad [66]$$

The solution for the value of this option P(V) is obtained in a similar way to the one shown in section[19], where the boundary conditions are:

$$\lim P_{V\to\infty} = 0 \quad [67a]$$
$$P(V^*) = L - \theta V^*(\theta) \quad [67b]$$

and V* reflects the value V_t at which it becomes optimal to abandon the investment; the functional form of P(V;θ) is easily obtained:

$$P(V;\theta) = \begin{cases} (L/(1+\beta))^{1+\beta}\,\beta^{\beta}\,(\theta V)^{-\beta} & \text{for } V > V^*(\theta) \quad [68a] \\ L - \theta V & \text{for } V \le V^*(\theta) \quad [68b] \end{cases}$$

where

$$V^*(\theta) = \frac{\beta}{1+\beta}\frac{L}{\theta} < L \quad [69]$$

and

$$\beta = \frac{-(\alpha-\sigma^2/2)+\sqrt{(\alpha-\sigma^2/2)^2+2r\sigma^2}}{\sigma^2} > 1 \quad [70]$$

Equation [69] shows the trigger value V*(θ) at which it becomes optimal to abandon given perfect coordination, where all agents get rid of their investments at a time $T^*(\theta)= \inf(t \geq 0: V(t) \leq V^*(\theta))$.
With this solution in hand, the analysis follows the one developed in section 4 taking into account that now the "leading agent" will be that with the lowest value for her private signal.

6 Conclusions

Investment and abandonment decisions can be analyzed by using the frame of the real options analysis. However, information regarding the true value of the parameters to be incorporated into the valuation of such options is not always accurately measured, giving rise to different estimations made by different agents (private signals). The existence of private signals and difficulties in the aggregation of information can give rise to invest before or after an optimal time. In a similar way, it can give rise to a behavior of a sequence of immediate exercise of real options even in the case where private signals are different among agents participating in the market. In the paper we have developed some conditions under which this behavior can be exacerbated, which is the case where there are few participant in the market, and the range of private signals is wide enough. Both conditions can be associated to underdeveloped markets, with not many participants, and prices not reflecting appropriately the information, giving rise to many private prices. If the case to be treated is the capital market, or the market for saving and investment, then investment decision can deviate from an optimal situation and sequence of investment decisions even with different sets of information may arise, giving rise to volatility in the amount of investment, and wide swings in the waves of investment and disinvestment aggregates. We left as a suggestion of future research to try to test the model.

[19] See Annex A from chapter III of Dapena (2004[a]).

References

Arrow, K. (1962). "The Economics Implications of Learning by Doing". *Review of Economics Studies* 29, 155-173.

Banerjee A. (1992), "A Simple Model of Herd Behavior". *Quarterly Journal of Economics* 107, 797-817

Black F., y Scholes M. (1973), "The Pricing of Options and Corporate Liabilities". *Journal of Political Economy* 81 (May-June): 637-659.

Caplin A. y Leahy J. (1994), "Business as Usual, Market Crashes and Wisdom after the Fact". *American Economic Review* Vol 84 Nro. 3.

Cox J., Ross, S., y Rubinstein M. (1979), "Option pricing: A simplified approach". *Journal of Financial Economics* 7, no. 3:229-263

Dapena J., (2004a). "Opciones Reales para Decisiones de Inversión y Abandono en Contextos Macro de Volatilidad del Producto con Extensión a Mercado de Capitales ". Tesis Doctoral Universidad del CEMA, no publicada.

Dapena J., (2004b). "Un Modelo de Relación entre Volatilidad en Tasas de Crecimiento del Producto y Volatilidad en Tasas de Crecimiento en el Precio del Stock de Capital". Presentado en la XXXIX Reunión Anual de la Asociación Argentina de Economía Política.

Dixit A. y Pindyck R. S. (1994), *Investment under Uncertainty*, Princeton University Press, Princeton, N.J.

Ellison G. y Fudenberg D. (1993), "Rules of Thumb of Social Learning". *Journal of Political Economy* 101, 612-643.

Gale D. (1996), "What have we learned from Social Learning?". *European Economic Review* 40, 617-628.

Grenadier S. (1999), "Information Revelation through Option Exercise". *The Review of Financial Studies* Vol 12 Nro 1 95-129.

Grenadier S. (2000), *Game Choices: The Intersection of Real Options and Game Theory*. Risk Publications

Ingersoll J. (1987), *Theory of Financial Decision Mak*ing, Studies in Financial Economics. Rowman & Littlefield Publishers inc.

Kogut B. y Kulatilaka N., (2001), "Capabilities as Real Options". *Organization Science* Vol 12 Issue 6.

Kulatilaka N. y Marcus A. (1992), "Project valuation under Uncertainty: when does DCF fail?". *Journal of Applied Corporate Finance* 5, no. 3: 92-100

Kulatilaka N. (1995ª), "The Value of Flexibility: A Model of Real Options". *In Real Options in Capital Investment*. Ed. L. Trigeorgis. Praeger.

Mc Donald R. y Siegel D. (1984), "Option Pricing When the Underlying Asset Earns a Below-Equilibrium Rate of Return: A Note". *Journal of Finance* (March), 261-265

Mc Donald R. y Siegel D. (1985), "Investment and the Valuation of Firms When there is an Option to Shut Dow". *International Economic Review* 26 (June), 331-349

Mc Donald R. y Siegel D. (1986), "The Value of Waiting to Invest". *Quarterly Journal of Economics* (November) 101, 707-728

Merton R. C. (1973), "Theory of Rational Option Pricing". *Bell Journal of Economics and Management Science* 4, no. 1: 141-183.

Myers S. (1977), "Determinants of Corporate Borrowing ". *Journal of Financial Economics* 5.

Stiglitz J. y Weiss A. (1981), "Credit Rationing in Markets with Imperfect Information", *American Economic Review* 71 (3): 393-410.

Trigeorgis L. (1988), "A Conceptual Options Framework for Capital Budgeting". *Advances in Futures and Options Research* 3:145-167.

Trigeorgis L. (1997), *Real Options: Managerial Flexibility and Strategy in Resource Allocation.* The MIT Press, Cambridge Massachussets.