Job market signals and signs*

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Abstract

What happens to job market signaling under two-dimensional asymmetric information? With 2 types of productivity and noise, the equilibrium remains separating if an extended single-crossing condition is satisfied. If not, there are partially pooling equilibria where only extreme types can be distinguished, and supplementary information is needed. On-the-job interaction gives employers private information on productivity, which employment relationships may reveal to the market. While sticky wages lead to public revelation of this private information through dismissals, flexible wages do not, allowing employers to do cream skimming. Beyond the 2x2 case, employment relationships are always a noisy sign, so education is valuable as a life-time job market signal for high-ability workers.

Key words: two-dimensional asymmetric information, private information, informational rents, single-crossing, signals, signs

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1 Introduction

We analyze the informative role of signals in the Spence (1973) job market model. In Spence (1973), education signals productivity because more productive individuals have lower costs of education. However, subjective signals may depend on factors other than quality differences.

We ask what happens to signals when individuals differ not only in ability but also in other personal traits. We specifically posit that the costs of signaling depend on the taste for study. However, the costs of signaling might differ for a host of other reasons that affect the utility cost of education, like differences in time preferences, in the income of parents, or in the desire to achieve social recognition. As long as these factors do not directly affect work productivity, they are simply noise from the point of view of firms.

Since the taste for study is part of personal preferences, this is private information that has to be inferred from actions, just like ability. Once this noise is taken into account, does education still act as a separating signal? With two types of ability and two types of taste for study, we show a separating equilibrium still exists under two-dimensional asymmetric information if an extended single crossing condition is satisfied. However, no separating equilibrium exists when it is violated. Rather, there are partially pooling equilibria in which the probability the worker is more productive is monotonically increasing in the signal. Though signaling is still somewhat informative, only extreme types can be told apart.

This two-dimensional asymmetric information framework with four types of agents, more or less productive workers with a taste for study or not, is complementary to Riley (2001). Riley considers an extension of the original Spence model where there are also four types of agents, because some less productive workers have relatively low signaling costs in terms of education, to analyze the consequences of introducing "noise". However, Riley's focus is on equilibrium refinements. His main point is that the intuitive criterion no longer selects a unique partially pooling equilibrium.[1] He goes on to analyze other equilibrium refinements to define out-of-equilibrium events, and emphasizes that, as in screening models, the distribution of types is key in determining the existence of a unique equilibrium.

[1]Since inefficient equilibria can not be ruled out, there are multiple partially pooling equilibria with either low education (less productive workers with high signaling costs in terms of years of formal education), or high education (productive workers, or the two types of less productive workers with low signaling costs).

We do not try to tackle the problem of coordinating among the equilibria. The point we try to make is different: if there is asymmetric information on other dimensions of workers' characteristics, making education a noisy signal that does not lead to a separating equilibrium, employers need supplementary information to sort out the productivity of workers. The signaling role of education might be specially important on entry to the job market, but later on employers could rely on the job-market record for more information. Previous work experience is empirically important in job interviews (Behrenz 2001)

In relation to types of information, Spence (1973) distinguishes between indices and signals. Indices are fixed attributes of job applicants, unalterable observable attributes such as race and sex. Since age does not change at the discretion of the individual, Spence also considers it an index. Signals are observable characteristics that are subject to the manipulation by the individual, of which education was singled out by Spence. As to other sources of information, employers get to know a worker through day to day contact at work. This generates private information that allows to assess a worker's type better. This information is neither a fixed characteristic, nor is it subject to the direct manipulation of the worker. From the point of view of the worker, it is an involuntary "sign" generated along the work career that indicates underlying characteristics. This private information will affect employment relationships. If a worker is dismissed, this may in turn act as an informational sign that reveals this information to the whole job market.[2]

Employment relationships are comparable to lending relationships in the credit market. The creditworthiness of small firms or individuals may only be privately known to the bank or lender that has carried out transactions with them and developed a relationship. This lending relationship generates private information. However, the very existence of a relationship, if it is observable, can act as a public sign to third parties of who is a good credit or not. Getting a credit card or a loan can act as a good sign, and other financial intermediaries may try to get these clients.[3] The same may happen with people that have continued employment with a given firm, though it is

[2]In terms of game theory, dismissals can be seen as signals insofar as they reveal the employer's private information about the employee's type. The distinction we draw is that employment relationships are not signals sent by the employee.

[3]In my personal experience, several credit card applications were turned down because of lack of a previous credit record. However, after a special promotion by American Express for university students, offers from commercial banks started piling up.

not obvious how much information these employment relationships actually reveal to outsiders.

In a sense, this approach links Spence's (1973) view on signals when workers have private information about themselves, with Waldman's (1984) approach where employers have private information about their employees. To explore this implication and separate the earlier and later job career, we embed the signaling game in a two-period framework. Gibbons and Katz (1991) provide the insight that in a dynamic setting employment relationships may be a sign to other firms of the quality of workers in the job market. In a setup where outside firms only observe wage policies, we show that these signs turn individual productivity into public information if wages are sticky, but under flexible wages productivity remains private information and employers enjoy an informational monopoly.

Section 2 first looks at the effects of two-dimensional asymmetric information on the signaling role of education. Section 3 shows how employment relationships may be, or not, a sign of underlying productivity according to whether wages are sticky or flexible. Section 4 concludes.

2 Education as a signal

Akerlof (1970) pointed to devices such as guarantees as a potential way to solve problems of asymmetric information. Since guarantees are less expensive for sellers of high-quality goods than for sellers of low-quality goods, in principle high-quality sellers will be more willing to provide a guarantee.

Spence (1973) showed the conditions under which a signal that is less costly for high-quality sellers may indeed lead to separating equilibria where they differentiate themselves from low-quality sellers, as well as the possibility of pooling equilibria where the two types cannot be distinguished. Our signaling model builds on Spence (1973), where education is used as a signal in the job market, abstracting completely from the contribution of education to human capital.

Spence (1973) introduced heterogeneity in ability, so some individuals have flatter indifference curves and are willing to go farther in terms of education for any given wage increase. In our setup, preferences can differ along two dimensions, ability and other idiosyncratic factors that affect the psychic costs of education, which for simplicity we refer to as the taste for study. Once there is heterogeneity in another dimension, this introduces noise that can make the signal less informative. Whether this affects the original Spence results will depend on what can be interpreted as the signal-to-noise ratio.

The players are workers and firms. The timing is that workers first decide the level of education, taking into account its informational role in the job market. Competitive firms then make their wage offers, based on the expected productivity of workers according to their education.[4]

2.1 Preferences

Let a workers' utility depend positively on wages w and negatively on the cost of education c,

$$U(w, e, \theta, \nu) = w - c(e, \theta, \nu). \tag{1}$$

In turn, the utility cost of education c depends on education e, where $e \geq 0$, worker's ability type θ, and idiosyncratic factors ν such as the taste for education.

In keeping with the original Spence model, the influence of the parameters θ and ν on the costs of education are given an extremely simple formulation,

$$c(e, \theta, \nu) = \frac{c(e)}{\theta\nu}, \tag{2}$$

where high ability θ and high taste for education ν both lower the costs of education, and $c\prime(e) > 0$ (in the figures below, we assume $c(e) = e^2$ for concreteness). These assumptions imply that the slope of the indifference curves in space (e, w) are flatter for more able individuals (higher θ), and for individuals fonder of education (higher ν):

$$\left.\frac{dw}{de}\right|_{\overline{U}} = -\frac{U_e}{U_w} \quad \Rightarrow \quad \left.\frac{dw}{de}\right|_{\overline{U}} = \frac{c\prime(e)}{\theta\nu}. \tag{3}$$

Firms are risk-neutral and maximize profits. Ability type θ determines the productivity level. Profits equal a worker's productivity minus wages:

[4]The behavior of competitive firms can be represented by a single player that minimizes a loss function given by the quadratic difference between wages and productivity (Fudenberg and Tirole 1991, chap. 11).

$$\pi = \theta - w. \tag{4}$$

From a firm's point of view, only factor θ matters, while factor ν is irrelevant for its profits. It will, however, introduce noise into the signal. In a setting with perfectly competitive markets, expected profits will be zero, so in expected value wages will equal productivity.

2.2 Worker heterogeneity

We assume that ability may be either low or high, $\theta \in \{\theta_1, \theta_2\}$, and taste for education may also be low or high, $\nu \in \{\nu_1, \nu_2\}$. Heterogeneity among individuals implies that there are four types of agents, as shown in Table 1. A key assumption in what follows is that ability and taste for education are not perfectly correlated (if they were, the actual types of agents would be reduced to two).

<please insert Table 1.Probability distribution in 2x2 case>

Let heterogeneity in taste be denoted by

$$h \equiv \nu_2 - \nu_1. \tag{5}$$

Denote by $\widetilde{h}$ the knife-edge case of heterogeneity that separate the intervals of what will be characterized below as high and low signal-to-noise ratios:

$$\theta_1(\nu_1 + \widetilde{h}) = \theta_2 \nu_1. \tag{6}$$

The case $h \in [0, \widetilde{h}]$ will correspond to a high signal-to-noise ratio where tastes vary relatively less than productivity.[5] The case $h \in (\widetilde{h}, H]$, for some positive $H > \widetilde{h}$, will correspond to a low signal-to-noise ratio in which tastes vary relatively more than productivity.

[5] In the knife-edge case $h = \widetilde{h}$, the indifference curves of types (θ_1, ν_2) and (θ_2, ν_1) are exactly superimposed on each other.

2.3 Single-crossing

If no signal were available, all workers would have a common level of zero education. In that case, firms would offer workers a wage equal to expected productivity, i.e. $w = E(\theta)$, where $E(\theta) = (p_{11} + p_{12})\theta_1 + (p_{21} + p_{22})\theta_2$. We now analyze what happens when a signal is available to differentiate workers.

In terms of the present notation, the original Spence model corresponds to $h = 0$. This case boils down to two types of workers, high and low productivity. Spence (1973) showed there are a continuum of separating equilibria. These equilibria can be characterized as perfect Bayesian equilibria. By the Cho and Kreps (1987) intuitive criterion, however, only the least cost separating signal, where the low productivity worker is just indifferent between studying or not, remains; this equilibrium coincides with the Riley outcome (Riley 1979).

There are also pooling equilibria in Spence(1973). These perfect Bayesian equilibria can be discarded applying the Cho-Kreps (1987) equilibrium dominance arguments: a competent worker has lower signaling costs, so it will be willing to deviate to levels of education higher than what any incompetent worker would ever pick.

We now show that the result that the Riley outcome is the unique perfect Bayesian equilibrium that satisfies the Cho-Kreps refinement generalizes to the interval $h \in (0, \widetilde{h}]$, which can be interpreted as an interval with a high signal-to-noise ratio. In this interval, tastes vary relatively less than productivity:

$$\frac{\nu_2}{\nu_1} \leq \frac{\theta_2}{\theta_1}. \tag{7}$$

With one-dimensional asymmetric information, the Spence-Mirrlees single-crossing condition asserts that the slope of indifference curves is decreasing in θ. This differential condition can be related to single crossing as an ordering of types in terms of θ (cf. Edlin and Shannon 1998). With two-dimensional asymmetric information, the marginal costs of signaling are given by the slope of the indifference curves in (3), which in our specification are inversely related to the product $\xi = \theta\nu$. An extension of the Spence-Mirrlees condition to a two-dimensional setup is as follows:

Definition 1 *Single-crossing in θ is satisfied under two dimensional heterogeneity if the slope of indifference curves in space (e, w) is (i) decreasing in*

θ, and (ii) the decrease in θ is always greater than with any change in ν.

Differentiation of (3) shows condition (i) is satisfied. As to condition (ii), given our multiplicative assumption about the utility function, the two dimensions can be projected over a one-dimensional interval. In the $2x2$ case, condition (ii) in terms of the ordering of types can be expressed as:

$$\theta_1\nu_1 < \theta_1\nu_2 \leq \theta_2\nu_1 < \theta_2\nu_2. \tag{8}$$

The relative variation in the second dimension is crucial in determining the ranking of the marginal costs of signaling. When (7) holds, more productive workers indeed have flatter indifference curves than less productive workers, so (8) is satisfied.[6]

If the extended single-crossing property is satisfied, beliefs $\mu(.)$ on worker productivity in a separating equilibrium will be given by

$$\begin{cases} e = 0 & \Rightarrow \quad \mu(\theta = \theta_1 | e = 0) = 1 \\ e = e^s & \Rightarrow \quad \mu(\theta = \theta_2 | e = e^s) = 1 \end{cases} . \tag{9}$$

For out-of-equilibrium values of education e, we assume a firm will expect productivity θ_1 if $e < e^s$, and productivity θ_2 if $e > e^s$. These beliefs determine the conditional probability a worker is productive for each observed level of education.

$$\begin{cases} 0 < e < e^s & \Rightarrow \quad \mu(\theta = \theta_1 | e) = 1 \\ e > e^s & \Rightarrow \quad \mu(\theta = \theta_2 | e) = 1 \end{cases} . \tag{10}$$

One can define e^s by picking as signal the least-cost level of education that will differentiate more and less productive workers, as Figure 1 shows.

<please insert Figure 1. Single crossing: Separating equilibrium>

The least cost separating signal is determined by the less productive worker with a high taste for study, at point A in Figure 1. At this point, worker type (θ_1, ν_2) is indifferent between getting a high wage $w = \theta_2$ with

[6]The Appendix discusses condition (ii) for the NxN case of a finite number of types. With a continuum of types θ and ν, condition (ii) is impossible to fulfill unless the range of variation of ν is null.

education $e = e^s$, and a low wage $w = \theta_1$ with education $e = 0$. It is consistent with a Nash equilibrium to assume a worker will not signal when it is just indifferent (to break indifference, it would suffice to consider a signal $e^s + \epsilon$, with $\epsilon > 0$ that is arbitrarily small). More productive workers strictly prefer to signal and get a high wage, rather than not signal and get a low wage.

Given our specification for out-of-equilibrium beliefs, neither type of worker has an incentive to choose any other level of education because it would strictly reduce its utility. Hence, behavior will conform to (9), so this is indeed a separating equilibrium. The inefficient separating equilibria with levels of education $e^s > e$ can be discarded by application of the Cho-Kreps intuitive criterion.[7]

A pooling equilibrium $w = E(\theta)$, where all workers are paid the average productivity of the pool of workers, can be discarded as shown in Figure 2. This pooling equilibrium implies that, whatever the level of education, firms will infer that expected productivity is $E(\theta)$. However, the farthest that a low productivity worker is willing to deviate is point B, with education e^d. High productivity workers have lower signaling costs, so they can be better off to the right of that point. Since those deviations are dominated in equilibrium for less productive types but not for more productive types, by the intuitive criterion firms can infer that a worker has high productivity if levels of education larger than (or equal to) e^d are observed. That restriction on out-of-equilibrium beliefs destroys any pooling equilibrium.

<Please insert Figure 2. Single crossing: No pooling equilibrium>

Likewise, one can discard partially pooling equilibria where some of the types are bunched together, namely: the three types with highest ξ choose the same high signal, or the two intermediate types of ξ choose the same intermediate signal, or the three types with the lowest ξ pick the same low signal. The reason is that the indifference curves of more productive workers are flatter than the indifference curves of less productive types, so more productive workers will always be willing to deviate farther to the right than less productive workers to signal their type.

[7] There are a multiplicity of separating equilibria with more education than e^s. However, these Perfect Bayesian Equilibria do not satisfy the intuitive criterion: the out-of equilibrium beliefs would imply that low productivity workers can signal with positive probability in the interval to the right of e^s to get a high wage, when in fact that is dominated in equilibrium by not signaling and getting a low wage. Only high productivity workers will be willing to pick signals in that interval to get a high wage.

These results can be summarized as follows.

Proposition 2 *If single-crossing under two-dimensional heterogeneity holds, there is a unique perfect Bayesian equilibrium that satisfies the intuitive criterion. This is a separating equilibrium where less productive workers pick zero education and more productive workers pick a positive level of education that is just enough to signal their type.*

Hence, with a high signal-to-noise ratio the signaling results of the basic Spence model are robust to two-dimensional asymmetric information. In this interval, only the Riley outcome –the undominated separating equilibrium– survives refinements of the perfect Bayesian equilibrium that apply the intuitive criterion.

2.4 No single-crossing

In the interval $h \in (\widetilde{h}, H]$, condition (7) is no longer satisfied and the ranking of the two intermediate types is inverted:

$$\theta_1\nu_1 < \theta_2\nu_1 < \theta_1\nu_2 < \theta_2\nu_2. \tag{11}$$

More productive workers no longer have lower costs of signaling. Once the extended single-crossing condition does not hold, no separating equilibrium exists. Why not is easy to see from Figure 3: a worker of type (θ_2, ν_1) is not willing to go farther than point C in Figure 3, while a worker of type (θ_1, ν_2) is. That is, when the ranking is inverted, a less productive worker with high taste for study is willing to invest in more education than a productive worker with low taste for study.

<please insert Figure 3. No single crossing: No separating equilibrium>

A pooling equilibrium can be discarded as before by application of the intuitive criterion: a productive worker of type (θ_2, ν_2) will always be willing to deviate. A partially pooling equilibrium where type (θ_1, ν_1) worker picks zero education and the other three types with highest ξ pick a common positive level of education can be ruled out by a similar argument. However, two other logical possibilities for partially pooling equilibria cannot be ruled out. First, intermediate types pick an intermediate level of education, while other types pick either zero or high education. Second, all types except (θ_2, ν_2) pick zero education.

First, consider a partially pooling equilibrium with three signals. Type (θ_1, ν_1) picks zero education, $e = 0$. Intermediate worker types (θ_1, ν_2) and (θ_2, ν_1) send the same intermediate signal e^i. Finally, type (θ_2, ν_2) picks the highest level of education e^s. Beliefs $\mu(.)$ are given by:

$$\left\{ \begin{array}{ccc} e = 0 & \Rightarrow & \mu(\theta = \theta_1 | e = 0) = 1 \\ e = e^i & \Rightarrow & \mu(\theta = \theta_1 | e = e^i) = \frac{p_{12}}{p_{12}+p_{21}} \\ e = e^s & \Rightarrow & \mu(\theta = \theta_2 | e = e^s) = 1 \end{array} \right. . \tag{12}$$

For out-of-equilibrium levels of education, we assume expected productivity equals that of the lowest level of education within each interval. Out-of-equilibrium beliefs are:

$$\left\{ \begin{array}{ccc} 0 < e < e^i & \Rightarrow & \mu(\theta = \theta_1 | e) = 1 \\ e^i < e < e^s & \Rightarrow & \mu(\theta = \theta_1 | e) = \frac{p_{12}}{p_{12}+p_{21}} \\ e > e^s & \Rightarrow & \mu(\theta = \theta_2 | e) = 1 \end{array} \right. . \tag{13}$$

Expected productivity if an individual has intermediate education is $E[\theta | e = e^i] = \frac{p_{12}\theta_1 + p_{21}\theta_2}{p_{12}+p_{21}}$. The equilibrium with the least-cost signals e^i and e^s is represented graphically in Figure 4, where $\theta^i \equiv \frac{p_{12}\theta_1 + p_{21}\theta_2}{p_{12}+p_{21}}$.

<please insert Figure 4. No single crossing: Partially pooling equilibrium>

In the spirit of the Riley outcome, let the least-cost intermediate signal be determined at point D, with education $e = e^i$ and average wage $w^i = \theta^i$, on the indifference curve of type (θ_1, ν_1) that goes through point $e = 0$ and $w = \theta_1$. And let the least-cost high signal be determined at point E, where type (θ_1, ν_2) is just indifferent between point D and education $e = e^s$ with wage $w = \theta_2$. It is easy to show that no type will want to deviate. Given these levels of expected productivity, firms will be willing to actually pay these wages. However, this partially pooling equilibrium is not unique.[8]

[8] Cho and Kreps (1987) remark for the Spence signaling model with three types of productivity that the intuitive criterion is not always strong enough to ensure the Riley outcome. Similarly, here an intermediate signal in the range between point D in Figure 4 and the point where the indifference curve of type (θ_2, ν_1) through coordinates $(0, \theta_1)$ cuts the intermediate wage line (call it D') may also satisfy (12), with the high signal now determined where the indifference curve of type (θ_1, ν_2) through point D' cuts the high wage line (call this point E'). The only equilibria that can be ruled out over the interval $[D, D']$ are those where the highest possible wage for type (θ_1, ν_1), $w = \theta_2$, is at

Second, for some parameter values there might be another partially pooling equilibrium where the three types with lowest ξ pick zero education, and type (θ_2, ν_2) picks a positive level of education e^s.[9] Let equilibrium beliefs be:

$$\text{(14)} \qquad \begin{cases} e = 0 \quad \Rightarrow \quad \mu(\theta = \theta_1 | e = 0) = \frac{p_{11}+p_{12}}{p_{11}+p_{12}+p_{21}} \\ e = e^s \quad \Rightarrow \quad \mu(\theta = \theta_2 | e = e^s) \end{cases} .$$

For out-of-equilibrium levels of education, we assume expected productivity equals that at the lowest level of education within each interval. Out-of-equilibrium beliefs are:

$$\text{(15)} \qquad \begin{cases} 0 < e < e^s \quad \Rightarrow \quad \mu(\theta = \theta_1 | e) = \frac{p_{11}+p_{12}}{p_{11}+p_{12}+p_{21}} \\ e > e^s \quad \Rightarrow \quad \mu(\theta = \theta_2 | e) = 1 \end{cases} .$$

Expected productivity if an individual has no education is $E[\theta | e = 0] = \frac{(p_{11}+p_{12})\theta_1 + p_{21}\theta_2}{p_{11}+p_{12}+p_{21}}$. The partially pooling equilibrium with the least-cost signal e^s can be constructed by an argument similar to Figure 4. This is a perfect Bayesian equilibrium because no type of worker is willing to deviate. Additionally, there are other equilibria that are less efficient, so it is not unique equilibrium.[10]

The intuitive criterion is not always capable of ruling out partially pooling equilibrium (14). Let $\theta^{low} \equiv \frac{(p_{11}+p_{12})\theta_1 + p_{21}\theta_2}{p_{11}+p_{12}+p_{21}}$, and $\theta^i \equiv \frac{p_{12}\theta_1 + p_{21}\theta_2}{p_{12}+p_{21}}$. It is possible to rule out this equilibrium if the education-wage pair (e^d, θ_2) that leaves type (θ_1, ν_1) indifferent to education-wage pair (0, θ^{low}) is such that type (θ_2, ν_1) prefers (e^d,θ^i) to alternative (0, θ^{low}). Figure 5 illustrates the case when it is possible to rule out this equilibrium. This will depend on

an education level that leads to a point below the indifference curve that gives this type its equilibrium payoff. In that case, equilibrium dominance arguments can be used to rule out this type. However, since indifference curves of type (θ_1, ν_1) do not become vertical at D, there always remains a non-empty interval to the right of D over which equilibrium dominance arguments have no bite.

[9] I thank Gustavo Maradona for pointing this out.

[10] Namely, it is possible to have equilibria where the three types with lowest ξ pick some positive level of education, together with out-of-equilibrium beliefs that assign individuals with no education low productivity. These socially less efficient equilibria cannot be discarded by the intuitive criterion.

specific parameter values, requiring sufficiently large θ_2 or sufficiently small $\frac{p_{12}}{p_{12}+p_{21}}$.

<please insert Figure 5. No single crossing: Alternative partially pooling equilibrium?>

Consequently, we have established the following result:

Proposition 3 *If single-crossing under two-dimensional heterogeneity does not hold, multiple perfect Bayesian equilibria satisfy the intuitive criterion. There is a partially pooling equilibrium where types with low θ and ν pick zero education, intermediate types pick intermediate education, and types with high θ and ν pick high education; besides an undominated equilibrium, partially pooling equilibria with excess education are possible. For some parameter values there are partially pooling equilibria where types with high θ and ν pick high education, while the rest pick low education.*

Proposition 2 implies that extreme signals are still effective in conveying a workers' type. It is in the middle ground (which may include all but the most able and motivated employees) that there is noise and imperfect revelation of type. From the viewpoint of firms in our model, the parameter ν basically introduces noise into the signal. The setup without single-crossing can be interpreted as a case of a low signal-to-noise ratio.

As to the relevance of a partially pooling equilibrium, the Appendix analyzes the extended single-crossing condition in the NxN case: for a given range of variation of productivity, as the number $N-2$ of intermediate productivity types grows, it becomes impossible to satisfy single-crossing unless the range of variation in the second dimension shrinks faster (and disappears in the limit). How serious the issue of noise is will depend on the relative range of variation of each dimension: perhaps only close productivity types are bunched together, or instead very distant productivity types are.

3 Employment relationships as signs

If education is indeed a noisy signal, signaling via education will lead to a partially pooling equilibrium. This information could be specially relevant to determine entry requirements (again, we are abstracting from the role of education in the buildup of human capital, that enhances productivity in itself). Afterwards, one would expect firms to use other types of information to sort productive and unproductive workers. In this regard, we explore the role of employment relationships.

The process of revelation of productivity at work takes time, so to incorporate this feature requires a minimum of dynamics. To incorporate the information generated in an employment relationship, we assume there are two periods. The first period represents the early work career, while the second period represents the later work career. A workers' utility depends on wages w^t in periods $t = 1, 2$, as well as on the cost of education c. The parameter $\delta \leq 1$ represents the discount factor, while $l \geq 1$ represents the length of the later work career in relation to the early work career (in case both have same length, $l = 1$).

$$U(w^1, w^2, e, \theta, \nu) = w^1 + \delta l w^2 - c(e, \theta, \nu). \tag{16}$$

In the first period, the worker has private information on its productivity and signals with a given education level. Firms then make job offers conditional on educational levels. After the first period has elapsed, the employer observes the worker's true productivity. Employment relationships may turn this private information into public information in the second period.

We analyze how the revelation of information is related to wage-setting. We consider two polar cases, sticky and flexible wages. As to their relevance, Gottschalk (2005) explains a lot of the evidence on nominal wage flexibility in terms of measurement error. Bewley (2002) reports that wage cuts, defined as the reduction in the pay of an employee continuing to work under unchanged conditions, is low according to surveys of employers from several countries, and is negligible in the few existing studies of company records.

The arrangements on sticky wages may depend on fairness considerations, as in the Akerlof and Yellen (1988) fair wage/effort hypothesis. Indeed, Bewley (2002) reports that surveys of business managers responsible for compensation policy show that employers avoid cutting pay because doing so would hurt morale and goodwill, and hence productivity. It may well be that different corporations follow different norms of fairness, so not all need apply the same policy. Nevertheless, if most notions of fairness consider wage cuts unfair and lead to a reduction of work effort, this could lead to a prevalence of sticky wages in most firms.

Even if nominal wages are mostly sticky, an inflationary environment helps to flexibilize real wages, which are the relevant variable for the dismissal decision. In this regard, Gibbons and Waldman (1999) mention several studies that show real wage decreases are not rare, though demotions are.

Rather than analyze wage decisions, we will focus on the choice of the employer among wage policies (the employer also follows an employment policy that determines employment relationships). That is, an employer will set a rule, conditional on a worker's productivity, that will determine whether the wage is changed or not in the second period. Though a given wage policy can be seen as an ex-ante commitment that defines what an employer will do when productivity is revealed, the firm is free to pick its best wage policy (not all rules may be equally credible). The informational requirements for other firms of observing the employer's wage policy are smaller than observing wage decisions, because they do not need to know the specific wages that each employee is receiving. Furthermore, if other firms observed each individual wage offers, this would provide additional information on employees that goes beyond information contained in employment relationship.

3.1 Sticky wages

Suppose that wages are sticky, so firms cannot reduce the wages of their employees. In the context of this restriction, a firm that maximizes profits might want to dismiss (lay off or fire) in the second period workers whose first-period contract stipulates a wage larger than their productivity. To not enter into the issue of the duration of unemployment spells, we will simply assume that either there is a sign of continued employment relationship or not.

In the second period, the timing is that the informed firm (the employer) decides its employment policy, setting a cutoff productivity level below which workers are dismissed. For those workers who qualify for a renewal of their contract, the employer determines a wage policy conditional on worker type (the employer can offer a wage hike). The outside firm (that represents the competive market) observes the employment policy and the wage policy, and takes this information into account when defining its wage policy. Finally, the employer observes true productivity, which will determine whether the worker is dismissed or not, and workers who are not dismissed have to decide between staying on the job or switching firms.

We can solve the game by backwards induction. Workers will accept the highest job offer they get (one can assume that if workers are indifferent, they do not switch jobs). It is immediate to see what happens if the first-period equilibrium is separating: first-period wages equal productivity of workers, so outside firms will be willing to pay that same wage. Employers will match that, and offer to renew the contracts of all workers.

On the other hand, if the first-period equilibrium is partially pooling as in (12), there is a signal of intermediate education that corresponds to a mix of productive and unproductive workers.[11] Figure 6 depicts this case. We condider three employment policies: dismiss all these workers (d, d), dismiss less productive workers θ_1 whose productivity is below their first-period wage (d, n), or not dismiss any workers (n, n).

$<$*please insert Figure 6. Employment and wage policies of informed and uninformed firms*$>$

The policy of dismissing less productive workers (d, n) will indicate that dismissed workers have low productivity, and the rest have high productivity because there are only two types of productivity (we show below that employer will not prefer alternative employment policies). The outside firm can condition its wage offer on whether the worker is dismissed or not: it will be willing to pay dismissed workers a low wage θ_1, and continuing workers a high wage θ_2. Consequently, the employer has to offer its more productive workers a second-period wage equal to their productivity. This means that the employer will make zero profits on its continuing employees. Thus, regardless of whether the first period equilibrium is separating or partially pooling, in the second period there is an equilibrium where high productivity workers get a high wage and low productivity workers a low wage equal to their productivity.

Proposition 4 *With 2 productivity types, if wages are sticky there is an equilibrium where employment relationships reveal an employer's private information on productivity, so wages equal productivity in the second period.*

We can now analyze the first-period equilibrium. The solution is straightforward if the second-period equilibrium reveals the employer's private information. The key observation is that, if in the second period wages depend on underlying ability that is fully revealed to the market, wages are independent of education. That is, education only affects wages in the first period. The first period equilibrium can thus be analyzed as in Section 2, where the

[11]For some parameter values, partially pooling equilibrium (14) with two signals is also possible. The analysis would be similar to that in text.

interpretation is now that the costs of education have to be compared to the benefits in the early job career (period one).

As to alternative employment polices, foreseeing that dismissals reveal productivity, employers could decide not to dismiss anybody. Indeed, as an alternative, the employer can keep all workers, paying them θ^i (if it payed more, in expected value it would lose money). The outside firm will be willing to match that, since it will make zero profits. The employer can also dismiss all workers, in which case the outside firm will be willing to pay them their expected productivity θ^i. Since any of the three employment policies in Figure 6 leads to zero expected profits, the employer is in principle indifferent among them. However, the equilibria where employment relationships do not reveal an employers private information on productivity are not credible ex-post.[12] Furthermore, beyond 2 productivity types, the employment policy of discriminating between low and high productivity workers will provide positive rents, so it will be strictly preferred by the employer to the other policies where expected profits are always zero.[13]

3.2 Flexible wages

What happens if firms can reduce the wages of employees who are found to have low productivity? In that case, the outside firm will not be able to distinguish high and low productivity workers by their employment relationships, because there is no need of dismissing unproductive workers.

The second period timing is that the employer first decides its wage policy. The employer can condition its wage offer on the productivity type each employee has. The second-period offer may be either larger or smaller than the first period wage since there are no restrictions on wage cuts. The unin-

[12]Ex-post, the employer may want to switch policies: if revealed productivity of its employees with intermediate education were below the first period wage, it would not want to keep them all; if revealed productivity were above the first period wage, it would not want to dismiss them all.

[13]If we analyzed employment and wage policies instead, the result would be pretty similar. There is an equilibrium where the employer dismisses less productive employees, and keeps more productive employees with a wage of θ_2, when outside firms offer to pay continuing workers θ_2 whatever the wage offer of the employer. Dismissing all workers is an equilibrium if outside firms offer θ_2 to all continuing workers. Not dismissing any workers, offering them all θ^i, is only an equilibrium if all workers switch to outside firms when wage offers match (an assumption opposite to that made in text). In all three equilibria, the employer makes zero profits ex-post.

formed firm does not observe the exact offer the informed firm makes to each employee, but it observes the wage policy, in terms of what wages are offered to each type of worker. On the basis of the employer's wage policy, outside firms make the employees an unconditional counteroffer, which they cannot condition on the worker's type because this is inside information. Finally, each type of worker has to decide between staying on the job or switching firms.

We can solve the game by backwards induction. Workers will accept the highest job offer they get. Again, it is immediate to see what happens if the first-period equilibrium is separating. Outside firms will be willing to pay the first-period wages, which equal productivity. The employer will match that, offering a renewal of contracts. We assume that workers who are indifferent do not switch jobs.

On the other hand, if the first-period equilibrium is partially pooling, there is a signal of intermediate education that corresponds to a mix of more and less productive workers. Less productive workers will face a wage cut, since the employer will not be willing to pay more than θ_1. However, the employer has more options for more productive workers θ_2: schematically, it can offer to raise the wage, to reduce it, or to maintain it at level of first-period wage $\theta^i = \frac{p_{12}\theta_1 + p_{21}\theta_2}{p_{12}+p_{21}}$. This is represented in Figure 7.[14]

<please insert Figure 7. Wage policies of informed and uninformed firms>

For simplicity, the representation is restricted to conditional strategies where the informed firm always offers low productivity workers a low wage. On the other hand, it can offer high productivity workers either the same wage θ^i as in the first period, a low wage θ_1 (the same logic will apply to any other low wage), or a high wage θ_2 (the same logic will apply to any other high wage). As to the unconditional strategies of uninformed firms, they can offer wages between θ_1 and θ^i (any wage higher than θ^i would lead them to lose money). For simplicity, we only represent the two endpoints.

In view of these logical possibilities, if the employer offers type θ_2 employees a raise above θ^i, uninformed firms will have an incentive to offer a low wage equal to θ_1, because at any wage equal to or lower than the expected productivity of the pool of workers with intermediate education they will only attract lemons, losing money. If the employer offers type θ_2 employees a wage reduction below θ^i, uninformed firms can offer all workers a wage that is slightly higher (up to θ^i), attracting the whole pool and still making a

[14]The analysis for the partially pooling equilibrium with two signals is similar.

profit. The last possibility is to offer type θ_2 employees the same wage θ^i as in period one. If we assume for simplicity that a worker does not switch jobs when indifferent (to not have to add ε to break tie), then uninformed firms have an incentive to offer a low wage equal to θ_1: at a wage equal to average productivity θ^i of pool they would only attract lemons, loosing money.

Given this maximizing behavior of uninformed firms, namely, offering θ^i to pool of workers if the employer offers more productive workers less than θ^i, and offering θ_1 if employer offers them θ^i or more, the informed firm has an incentive to pick the conditional strategy of offering type θ_1 workers a wage equal to their productivity θ_1, and type θ_2 workers the same wage θ^i as in the first period. Consequently,

Proposition 5 *With 2 productivity types, if wages are flexible employment relationships do not reveal an employer's private information on productivity. If the first period signaling game is separating, wages equal productivity in the second period. If the first period signaling game is partially pooling, the employer will have an informational rent on some high productivity workers, while other wages will equal productivity in the second period.*

The main insight from flexible wages is that, if employers can use conditional wage strategies, information on productivity may remain private even when there are only two types of productivity. The employer will have an informational monopoly: the informed firms will be under no pressure to raise the wage of productive workers with intermediate education. Hence, the informed firm can engage in cream-skimming, paying more productive workers with less education below their full productivity. Given this second period outcome, wages for productive workers in the second period will not be independent of education.[15]

[15] Our analysis differs from Gibbons and Katz (1991), because we consider wage policies instead of wage offers, and two productivity types instead of a continuum of types. For wage offers with two productivity types, one can find the following equilibrium: the employer offers a wage θ_1 to θ_1 type workers, and plays a mixed strategy of offering θ_2 type workers either a wage of θ_1 or a wage larger than θ_1 (which can be as large as θ_2). The uninformed firm offers a wage equal to expected productivity of pool of workers that receive a wage offer of θ_1, and a wage θ_2 to workers that receive any wage offer larger than θ_1. Given these strategies, both the employer and the uninformed firms are making zero profits, and neither has an incentive to deviate. Hence, with 2 productivity types, if uninformed firms observe individual wages there will be no rents, instead of positive rents when they only observe wage policy. This makes sense because individual wages convey

This result implies that the first and second period games cannot be analyzed separately when productivity remains private information. If a productive worker invests in more education in the first period, it can earn higher wages not only in the first period, but also in the second period. On the other hand, for unproductive workers education only leads to higher wages in the first period. I.e., for more productive workers $w^2 = w^1$, while for less productive workers $w^2 = \theta_1$. Hence, the objective functions in the first period can be simplified as follows:

$$\left\{ \begin{array}{c} \theta = \theta_2 : U(w^1, w^2 = w^1, e, \theta_2, \nu) = w^1(1 + l\delta) - c(e, \theta_2, \nu) \\ \theta = \theta_1 : U(w^1, w^2 = \theta_1, e, \theta_1, \nu) = w^1 + l\delta\theta_1 - c(e, \theta_1, \nu) \end{array} \right. . \tag{17}$$

This implies that the indifference curves of more productive workers become flatter in the first period:

$$\left\{ \begin{array}{c} \theta = \theta_2 : \frac{dw_1}{de}\Big|_{\overline{U}} = \frac{c\prime(e)}{(1+\delta l)\theta_2\nu} \\ \theta = \theta_1 : \frac{dw_1}{de}\Big|_{\overline{U}} = \frac{c\prime(e)}{\theta_1\nu} \end{array} \right. . \tag{18}$$

The high signal-to-noise case of Section 2 becomes more likely because more productive workers are willing to go farther to invest in education: the educational signal affects their earnings over their whole work career, in contrast to less productive workers who only benefit in their early work career. Though signaling becomes more revealing, a partially pooling equilibrium is not ruled out if $(1 + \delta l)\theta_2/\theta_1 < \nu_2/\nu_1$.

The first-period analysis has to be amended for a feature similar to Waldman (1984): given our assumption of perfect competition, all workers with intermediate education will be paid a bond in the first period equal to the discounted value of the informational rent that employers enjoy. This bond is shared by more productive workers with intermediate education and less productive workers with the same education. The bond b^i, discounted at interest rate r, equals:

$$b^i = \frac{p_{21}}{p_{12} + p_{21}} \frac{\theta^2 - \theta^i}{1 + r}. \tag{19}$$

more information about individual worker than wage policies do. However, the feature that θ_2 type workers receive a wage below their productivity remains.

This bond b^i has to be reckoned with in the computation of the first period equilibrium: type (θ_2, ν_1) has to forego this bond if it switches from intermediate signal e^i to high signal e^s, while type (θ_1, ν_2) gets this bond if it switches from low signal $e = 0$ to intermediate signal e^i. Hence, in some borderline cases there may be a partially pooling equilibrium, instead of a separating equilibrium, because of this bond.

3.3 Empirical implications

Under sticky wages, Table 2 shows there is a positive correlation between education and wages: more highly educated workers on average get higher wages in the second period. Hence, even if education is a noisy signal, the standard implication of Spence (1973) stands over time. Second, there is a positive correlation between employment relationships (workers who have not been dismissed) and wages. That is, dismissed workers earn less, and this effect is important for people with more education. This is precisely the empirical pattern of layoffs and lemons so nicely studied by Gibbons and Katz (1991).

<*please insert Table 2. Second period wages with public information*>

Under flexible wages, Table 3 shows that there is also a positive correlation between education and wages: more highly educated workers on average get higher wages in the second period ($w^{1,i}$ stands for the wage that individuals with intermediate education receive in periods 1 and 2). However, workers with the same productivity and different education may earn different wages, due to the informational rents that employers enjoy.

<*please insert Table 3. Second period wages with private information*>

If the first period were only a short probationary period where the employer could gather all the relevant information on productivity, education would not be very relevant as a job-market signal. For the present framework to add empirically relevant insights, the early work career has to be relatively extensive in relation to the later work career. In this regard, a partially pooling equilibrium with three signals in the first period implies a growth over time of the variance of wages for higher educational levels, because in the second period some workers with intermediate education get high wages and others get low wages.[16] This can be related to Mincer (1974),

[16]This would not happen in the partially pooling equilibrium with two signals. However, since for some parameter values this equilibrium does not exist, in the text we concentrate

who has a table based on 1960 U.S. Census (reproduced in Weiss 1983) that shows that while the variance of the log of weekly earnings hardly changes for people with 5-8 years of schooling, for people with 12 years of schooling it rises smoothly but steadily, from .205 at ages 24-29 to .317 at ages 55-59. And for people with 16 years of schooling, where signaling can be expected to be especially important because of the more autonomous nature of jobs, the variance rises a lot: from .235 for 24-29 year-old group and .277 for 35-39 year-old group, to .424 for 45-49 year-old group, and .552 for 55-59 year-old group. Albeit indirectly, the fact that the variance in earnings for people with 16 years of schooling at first does not rise much might indicate that the early work career could represent a period of up to 10 or 15 years.

Ashenfelter and Krueger (1994) find that returns to education for identical twins may be as large as for the population as a whole, which in the line of human capital models can be interpreted as isolating the effect of education on earnings, controlling for ability. Weiss (1995) also offers a sorting explanation: firms will infer a worker's unobserved ability from the educational choice, so education should affect initial wages according to the signaling model. Weiss goes on to say that if the signaling model is correct, the return to schooling across twins should decline over time, and he finds some evidence in that direction. However, the results above show that the implication of the signaling models critically depend on whether the employer's private information becomes public or not: if it becomes public, wage differences should decline over time, but if it doesn't, wage differences will not decline.[17] The implication of private information is that wages need not match productivity, because the employer's private information leads to informational rents. In this context, education as a signal will not only affect the wages of more able workers in their early job career, but rather over their whole career.

In the 2x2 case, productivity becomes public information with sticky wages regardless of education in the first period, while with flexible wages productivity remains private information for intermediate types. If sticky wages are prevalent, an implication is that there should be less dismissals under an inflationary environment than under stable prices, because inflation

on the partially pooling equilibirum with three signals.

[17]In fact, the model with flexible wages implies that difference will increase, because workers with intermediate education are paid an extra bond in the first period equal to expected discounted value of the informational rent in the second period. Wages that decline over the life-cycle are counter-factual, but we are abstracting from the positive influence of experience and human capital on earnings.

is a real-wage reducing device. This may also be a specific example in which inflation can contribute to make prices (here, wages) less informative.

When there are more than two productivity types, the Appendix shows that the result on full revelation of employer's private information through employment relationships under sticky wages does not hold. This is not surprising: in a different setup, Gibbons and Katz (1991) show that with a continuum of productivity types, dismissals only reveal that workers with a continuing employment relationship have productivity above a certain cut-off level. What is true is that more information is always revealed under sticky wages than under flexible wages. The key intuition is that uninformed firms are always willing to pay the pool of continuing workers their expected productivity, which exceeds first-period wage under sticky wages, but equals first-period wage with flexible wages. The higher wage offers that uninformed firms are willing to make under sticky wages limit the informational rents of the informed firm.

The fact that on-the-job productivity does not fully become public information might help explain the role of an MBA, where people with work experience enroll. In comparison to other graduate programs, an MBA has less of a human capital role, and more of a signaling role. Indeed, some participants in the process describe it as basically going around from one meeting to another with corporate representatives. Such a limited role can make perfect sense to those students that basically need the MBA as a system-wide signal to reveal their ability, if their work record does not accomplish that for them. By (18), even if ability did not reduce the costs of education, high productivity agents would still have lower signaling costs because education increases their life-time earnings. This leads to self-selection: only people who believe they are high-ability have a large incentive to invest in an MBA as a job-market signal.

4 Conclusions

This paper analyzes the implications of two-dimensional asymmetric information for Spence's (1973) signaling model. Both ability and an idiosyncratic factor, called taste for study, can be either high or low. An extended single-crossing condition is satisfied when the ranking of signaling costs is basically determined by ability. When there is single-crossing, the equilibrium is separating. When not, the equilibrium becomes partially pooling.

The second dimension basically allows to analyze the influence of noise on the informative content of the signal. If education is indeed a noisy signal, it will be an imperfect proxy for a worker's productivity so other information will be used by firms. In this regard, the work record is singled out in this paper. Interaction at the work place generates private information on productivity for the employer. This is modeled in a dynamic setting where the first period, which represents the early job career, is the signaling game. The second period represents the later job career, where employers use information from the first period to judge productivity.

One way information on work productivity observed by the employer may become public is through the Gibbons and Katz (1991) idea on layoffs revealing lemons. Outside firms observe employment relationships. Continuity of working relationships can act as a sign of high productivity to other firms if higher productivity workers are more likely to keep their job. We find that if wages are flexible, employment relationships reveal no private information at all. Employers do not need to dismiss lemons, they can simply reduce their wages.[18] With sticky wages, on the other hand, employment relationships can be a sign to the market of which employees have high productivity, and dismissals indicate who are lemons.[19]

When the analysis is extended to consider more types of ability and taste for study, it becomes harder to satisfy the extended single-crossing condition that ensures education is a separating signal. Unlike the continuous case with asymmetric information in one dimension (Spence 2002), one can expect that with asymmetric information and heterogeneity in two dimensions, the equilibrium with a continuum of types will never be separating, but rather partially pooling. However, signaling is quite resilient to the introduction of asymmetric information in two dimensions, in the sense that average productivity is still increasing in the degree of education, and extreme types still send unequivocal signals. However, intermediate types will be difficult to tell apart.

As to the informative role of employment relationships, when one goes

[18]We analyze in text the case where outside firms observe the employers' wage policies. The result is similar when outside firms observe individual wages, but there wage offers convey in themselves additional information about the individual worker's productivity.

[19]The model completely ignores that work productivity is in part a matter of matching the right person to the right job (Jovanovic 1979). This increases the amount of asymmetric information, since a worker does not know its productivity type before hand. Dismissals will indicate a mismatch, but not necessarily that dismissed workers are lemons.

beyond the $2x2$ case revelation of information is always incomplete even if wages are sticky. Hence, employers will have an informational rent, paying some more productive workers less than their full productivity (under perfect competition, firms will in turn pay out a bond, which more able workers will have to share with the less able). Given incomplete revelation of private information, education becomes important for high ability workers as a signal not only in their early job career, but over their whole work cycle.

If education is a life-time signal only for individuals with high ability, studying can make perfect sense to them despite the fact they face a larger opportunity cost of not working. The higher opportunity cost is more than compensated by the larger earnings over the complete career, making signaling costs of high ability individuals smaller than those of low ability individuals. In other words, a prestigious degree can help to land a good job, but not to hold on to it, so this leads to self-selection when signaling.

A natural extension of this model is to combine the signaling role of education with the human capital role of education (Becker 1964), to derive more precise empirical implications about the complete effects of formal education and on-the-job training and experience. There are other signs at work besides employment relationships. For example, more able workers may be assigned more complicated jobs, or jobs with higher responsibility. If job hierarchies are visible to outside firms, this can act as a sign of work productivity. This is precisely what Waldman (1984) proposes: job assignments as a sign of productivity.[20]

Finally, the present framework considers noise as an example of two-dimensional asymmetric information. This issue is already present in the Akerlof (1970) lemons model: there is a problem with lemons because there are some dishonest sellers who are willing to misstate the quality of their car. It might be applied to consider the closely related issue of the influence of character on productivity. Though some traits of character like perseverance might also be captured by formal education (Weiss 1995), others will not.

[20]Waldman also makes the point that, with a continuum of types, job assignments will only reveal part of this private information to the market, i.e., that those assigned to the higher productivity job are above a certain ability level.

5 Appendix

5.1 Single-crossing in the NxN case

In the NxN case, if the log of θ_i and ν_j, $i, j = 1, 2, ..., N$ are evenly spaced, then

$$\ln \theta_i = \ln \theta_{i-1} + \frac{1}{N-1} \ln \frac{\theta_N}{\theta_1} \tag{20}$$

and

$$\ln \nu_j = \ln \nu_{j-1} + \frac{1}{N-1} \ln \frac{\nu_N}{\nu_1} \tag{21}$$

For the single-crossing condition under two-dimensional heterogeneity to be satisfied in the NxN case when the log of types in each dimension are evenly spaced, a necessary and sufficient condition is that:

$$\frac{v_N}{v_1} \leq \frac{\theta_N}{\theta_{N-1}} \tag{22}$$

It is necessary, because otherwise $\theta_{N-1} v_N > \theta_N v_1$ and there is at least one lower productivity type that is willing to go farther in terms of education than a higher productivity type. It is sufficient, because due to our assumption that the log of types is evenly spaced, $\theta_2/\theta_1 = ... = \theta_{N-1}/\theta_{N-2} = \theta_N/\theta_{N-1}$, which together with (22) implies that $\theta_1 v_N \leq \theta_2 v_1$, ..., $\theta_{N-2} v_N \leq \theta_{N-1} v_1$. Inequality (22) boils down to condition (7) for single-crossing in Section 2 when $N = 2$.

Inequality (22), under our assumption of evenly-spaced types, can be rewritten as

$$\ln \frac{v_N}{v_1} \leq \frac{1}{N-1} \ln \frac{\theta_N}{\theta_1} \tag{23}$$

For a given range of variation (v_N/v_1) in the second dimension, the single-crossing condition becomes increasingly harder to satisfy as N grows. Conversely, as N grows without limit, the range of variation of ν has to shrink to zero for single-crossing to be satisfied.

5.2 Employment relationships as noisy signs in $3x3$ case

Consider a $3x3$ example, with three types of productivity θ_i and three types of taste for study v_j, $i, j = 1, 2, 3$, where the types are evenly spaced apart so $\theta_2/\theta_1 = (\theta_3/\theta_1)^{1/2}$ and $\nu_2/\nu_1 = (\nu_3/\nu_1)^{1/2}$.

Assume that $v_3/v_1 > \theta_3/\theta_1$, so $v_2/v_1 > \theta_2/\theta_1$ and $v_3/v_2 > \theta_3/\theta_2$. This is only interesting scenario where there is a qualitative difference with the $2x2$ case; otherwise, there is no educational signal for which workers of three types of productivity can pool together.

Two orderings are possible, either $v_2/v_1 > \theta_3/\theta_1$, so ranking is basically determined by ν, i.e.,

$$\theta_1\nu_1 < \theta_2\nu_1 < \theta_3\nu_1 < \theta_1\nu_2 < \theta_2\nu_2 < \theta_3\nu_2 < \theta_1\nu_3 < \theta_2\nu_3 < \theta_3\nu_3, \tag{24}$$

or $v_2/v_1 \leq \theta_3/\theta_1$ so ranking is given by

$$\theta_1\nu_1 < \theta_2\nu_1 < \theta_1\nu_2 \leq \theta_3\nu_1 < \theta_2\nu_2 < \theta_1\nu_3 \leq \theta_3\nu_2 < \theta_2\nu_3 < \theta_3\nu_3. \tag{25}$$

We will analyze the second case, but the first case would be similar for our purposes.

The distribution of types detailed in Table 4 will determine how the different types group.

<please insert Table 4. Probability distribution in 3x3 case>

Without entering into a full characterization of possible equilibria, let e^i denote education level, with five levels, $e^0 < e^1 < e^2 < e^3 < e^4$, $e^0 = 0$, where each e^i is associated to the following expected productivity:

$$\left\{ \begin{array}{c} e = 0 \quad \Rightarrow \quad E[\theta|e=0] = \theta_1 \\ e = e^1 \quad \Rightarrow \quad E[\theta|e=e^1] = \frac{p_{12}\theta_1 + p_{21}\theta_2}{p_{12}+p_{21}} \\ e = e^2 \quad \Rightarrow \quad E[\theta|e=e^2] = \frac{p_{13}\theta_1 + p_{22}\theta_2 + p_{31}\theta_3}{p_{13}+p_{22}+p_{31}} \\ e = e^3 \quad \Rightarrow \quad E[\theta|e=e^3] = \frac{p_{23}\theta_2 + p_{32}\theta_3}{p_{23}+p_{32}} \\ e = e^4 \quad \Rightarrow \quad E[\theta|e=e^3] = \theta_3 \end{array} \right. \tag{26}$$

In a manner similar to Figure 4, one can define each successive e^i, $i = 1, 2, 3$, as the least-cost signal such that no type has an incentive to deviate. For this to be a perfect Bayesian equilibrium, expected productivity has to be strictly increasing in the signal. If not, either types grouped at e^3 would prefer to deviate to e^2, or types grouped at e^2 would prefer to deviate to e^1, or both. This would reduce the number of educational signals from five to either four or three, producing even more bunching of types. Other than this, the analysis would be similar to equilibrium (26).

In equilibrium (26), only the extreme type (θ_3, ν_3) goes far enough to single itself out with an educational signal. For all the types in between types (θ_3, ν_3) and (θ_1, ν_1), there will be some bunching (in the extreme, they will all pick the same signal), so it will not be possible to tell them perfectly apart.

Take the case where wages are sticky (if wages were flexible, we already know that employment relationships reveal no information at all). If the partially pooling equilibrium in the first period were given by (26), Will wages in the second period still be independent of education (as happened in Section 3)?

In the second period the employer will have an incentive to dismiss workers whose productivity is lower than their wage. This will perfectly reveal the worker's exact productivity for intermediate educational level e^2 (e^4): those dismissed have productivity θ_1 (θ_2), so those retained have productivity θ_2 (θ_3).

However, this sign does not work perfectly for educational level e^3. The expected productivity of workers with education e^3 is either above or below θ_2. If (i) it is equal to, or less than, θ_2, the employer will dismiss type θ_1 employees. This will reveal the type of dismissed workers, but not of continuing workers, of whom the market only knows that their expected productivity is

$$E[\theta|e = e^3, \textit{not dismissed}] = \frac{p_{22}\theta_2 + p_{31}\theta_3}{p_{22} + p_{31}}. \tag{27}$$

If the employer offers type θ_3 less than (27), the uninformed firms will find it profitable to attract the whole pool of continuing workers, offering an ε more (the analysis is similar to Figure 6). Hence, the employer will be willing to pay type θ_3 a wage equal to expected productivity of pool, namely (27), and type θ_2 a wage of θ_2. Given these conditional wage offers of the informed firm, the optimal response of uninformed firms is to offer θ_2 to the pool; if they offer more, they will lose money. This result implies that employment relationships will never allow the market to differentiate between type (θ_3, ν_1)

and type (θ_2, ν_2), so earnings of type (θ_3, ν_1) will depend on education in the long run.

What would happen if the expected productivity of types that pick e^3 were (ii) larger than θ_2? In that case, type (θ_2, ν_2) would be dismissed together with type (θ_1, ν_3), so outside firms would only be willing to pay pool of dismissed workers their expected productivity. For total revelation of productivity of the lower productivity types, one would need to add more rounds.

In either case, employment relationships only lead to partial revelation of the employer's private information. However, while (ii) has transitory consequences, (i) has permanent consequences because adding more periods would not change the state of affairs. Hence, even if wages are sticky, with three types of productivity dismissals no longer produce full revelation of types. This relates to the Gibbons and Katz (1991) results for a continuum of productivity levels.

To determine whether the first period equilibrium in a dynamic setup will indeed be as specified in (26), one has to incorporate the long-term value of education for more able workers in the first period signaling game. If the problem of noise is strong enough, it will still be possible to find an equilibrium for which there is bunching of workers with different productivity levels in the first period.

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Table 1. Probability distribution in $2x2$ case

		Taste ν	
		ν_1	ν_2
Ability θ	θ_1	p_{11}	p_{12}
	θ_2	p_{21}	p_{22}

Table 2. Second period wages with public information

	Wages w_2
Education	
- none	θ_1
- positive	$\{p_{12}\theta_1 + (p_{21} + p_{22})\theta_2\}/(p_{12} + p_{21} + p_{22})$
Employment relationship	
- dismissed	θ_1
- renewed	$\{p_{11}\theta_1 + (p_{21} + p_{22})\theta_2\}/(p_{11} + p_{21} + p_{22})$

Table 3. Second period wages with private information

	Wages w_2
Education	
- none	θ_1
- positive	$(p_{12}\theta_1 + p_{21}w^{1,i} + p_{22}\theta_2)/(p_{12} + p_{21} + p_{22})$
Ability	
- low	θ_1
- high	$(p_{21}w^{1,i} + p_{22}\theta_2)/(p_{21} + p_{22})$

Table 4. Probability distribution in $3x3$ case

		Taste ν		
		ν_1	ν_2	ν_3
	θ_1	p_{11}	p_{12}	p_{13}
Ability θ	θ_2	p_{21}	p_{22}	p_{23}
	θ_3	p_{31}	p_{32}	p_{33}

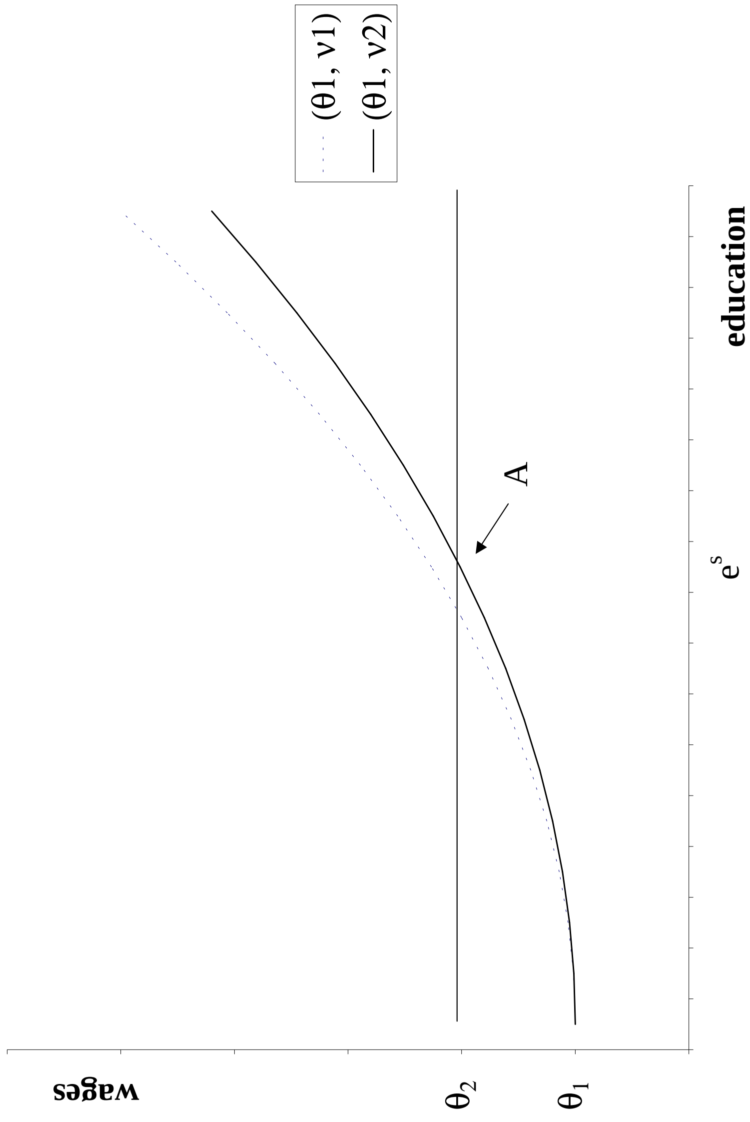

Figure 1: Single crossing: Separating equilibrium

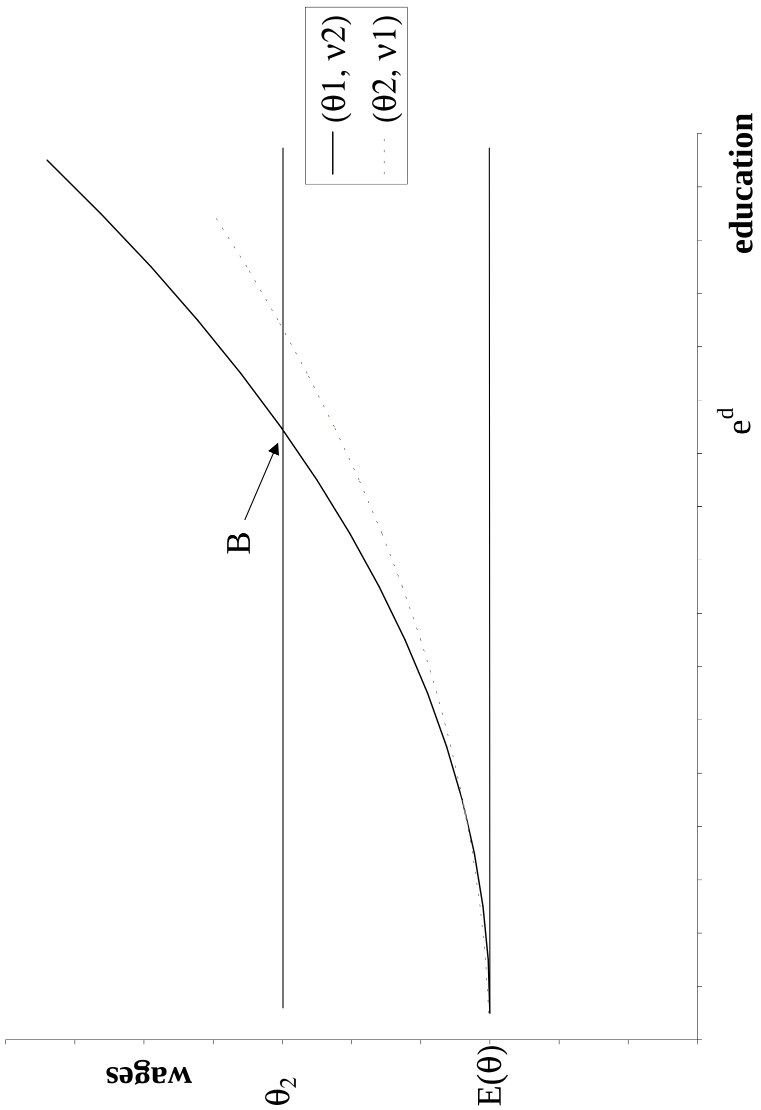

Figure 2: Single crossing: No pooling equilibrium

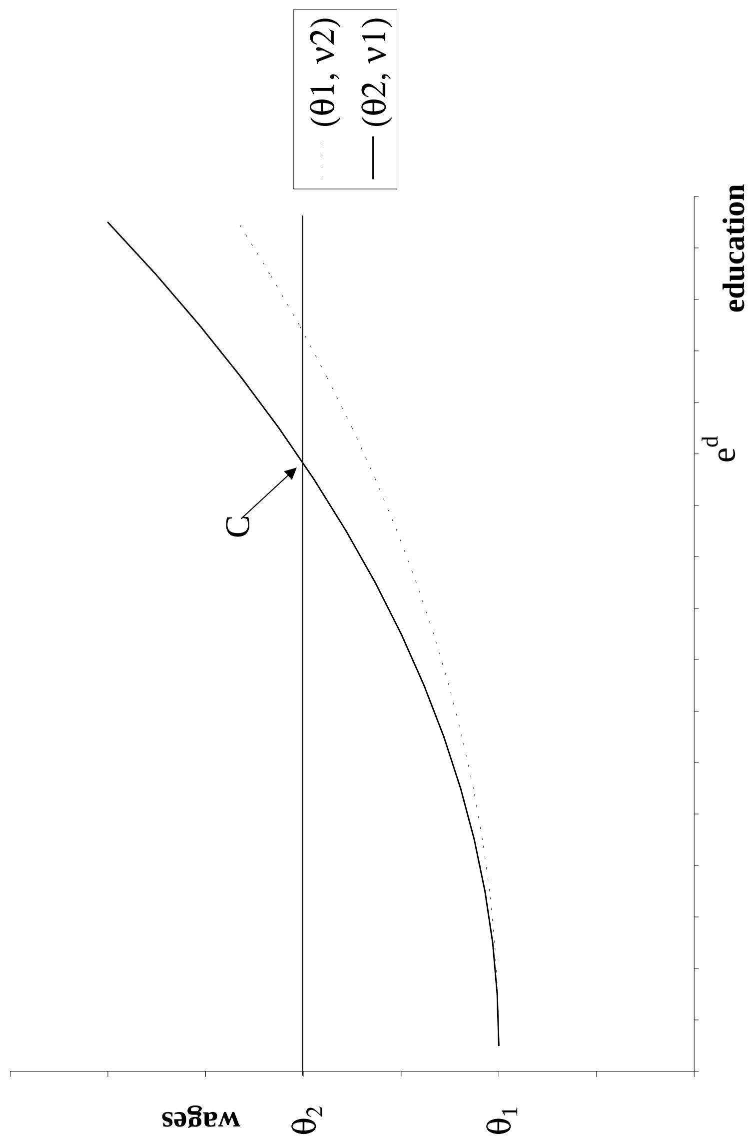

Figure 3: No single crossing: No separating equilibrium

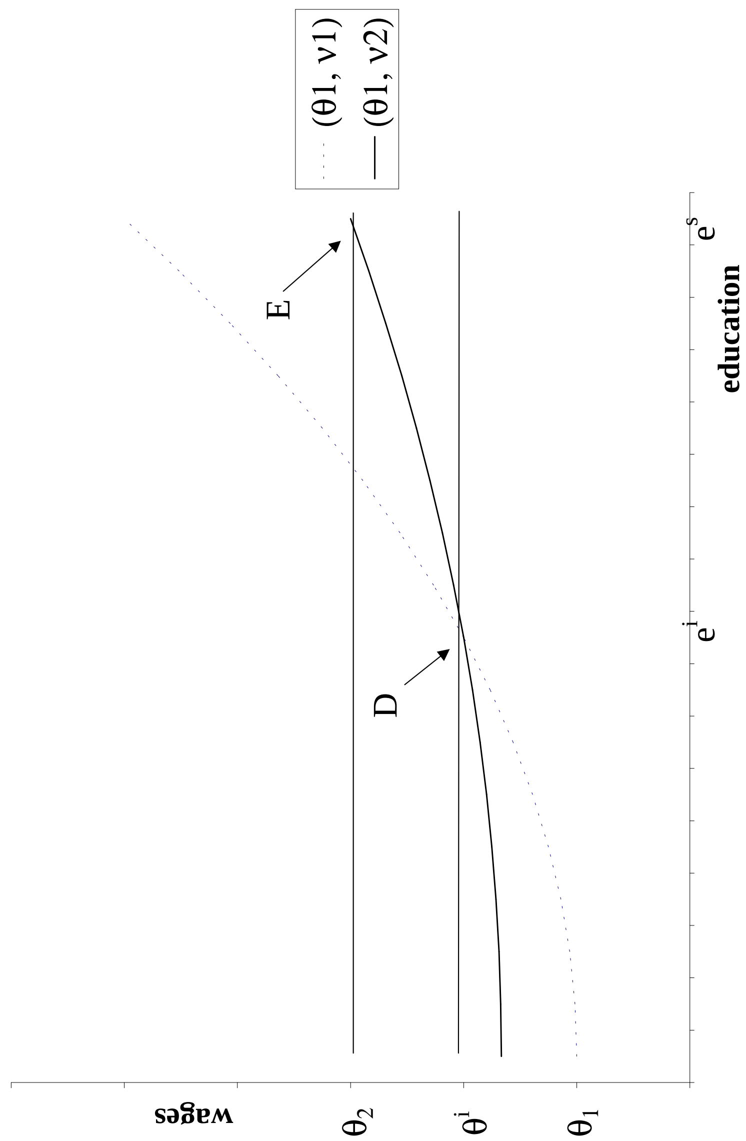

Figure 4: No single crossing: Partially pooling equilibrium

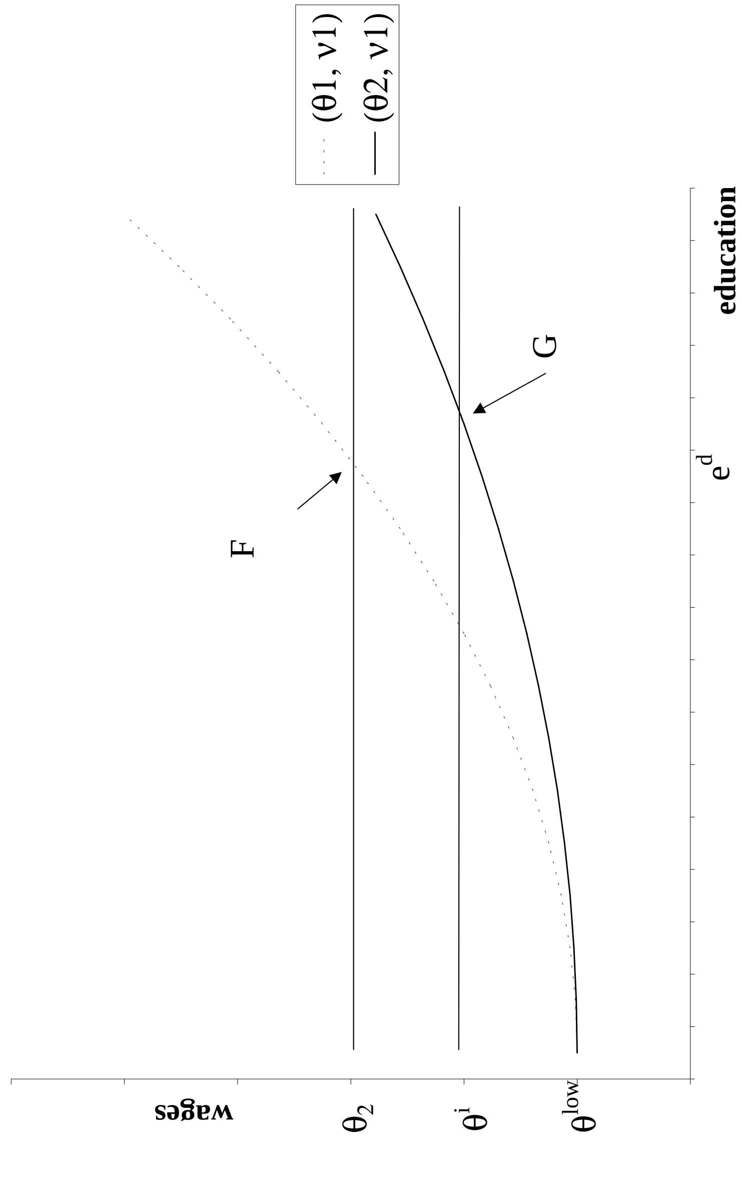

Figure 5: No single crossing: Alternative partially pooling equilibrium?

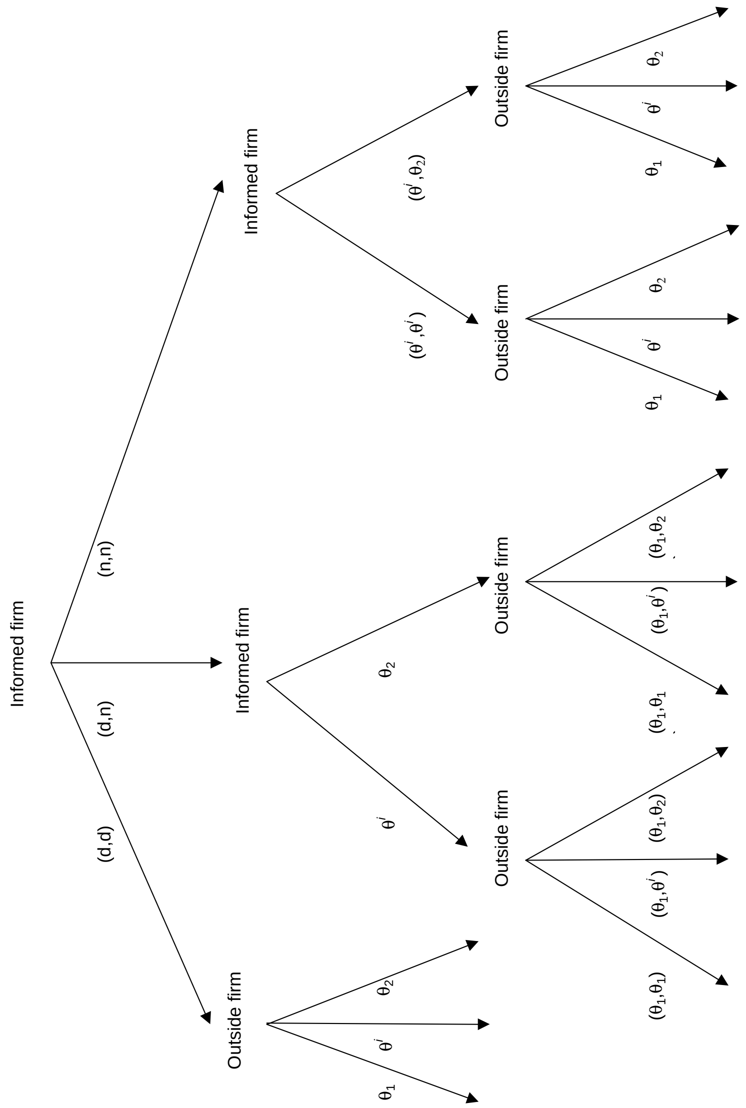

Figure 6: Employment and wage policies with sticky wages

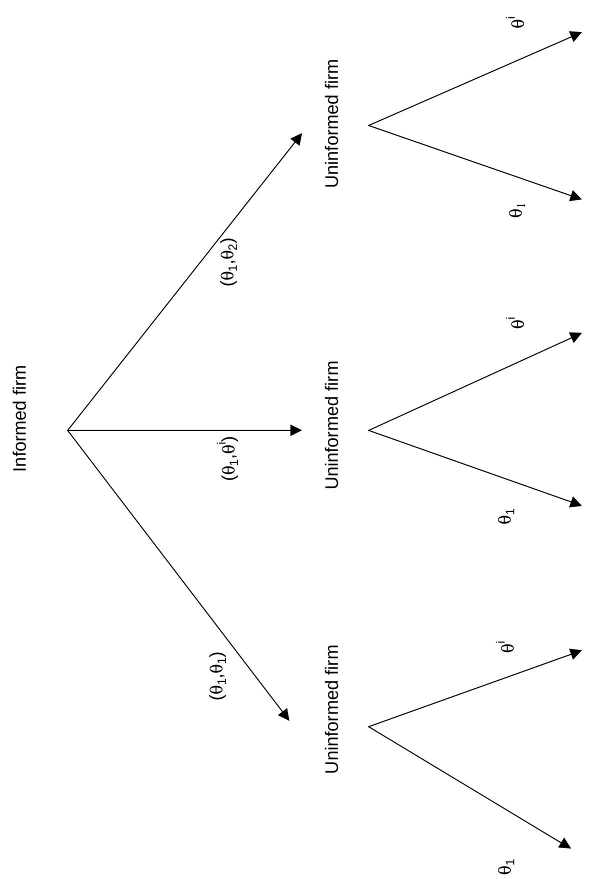

Figure 7: Wage policies with flexible wages