A REAL OPTIONS APPROACH TO TENDER OFFERS AND ACQUISITIONS PROCESSES

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Brief

Corporate control has added value for an investor since it gives degrees of freedom about the use of assets, sources of finance, salaries, etc. On the other hand, real options create value through the flexibility associated to the ability to react to some relevant uncertainty. The process of acquisition of corporate control can have two real options associated, a waiting option and a growth option. In the waiting option value is created through sequential investment instead of investing at once, while the growth option carries all the private benefits the investor can seize from control by making follow up investments, which can also justify premiums paid above the former market price. A relevant proposition of our paper is that the exercise price of the growth option (and hence the amount to be paid as the control premium) can be affected by the release of information. We develop a model for these two theoretical extremes, one where the exercise price fully reacts to events, and one where the exercise price does not react at all, and we obtain that the timing of the process of acquiring control would depend on the reaction of the price to be paid to obtain control, so would the size of the control premium over the former price.

Fields **: G13, G31, G34**.

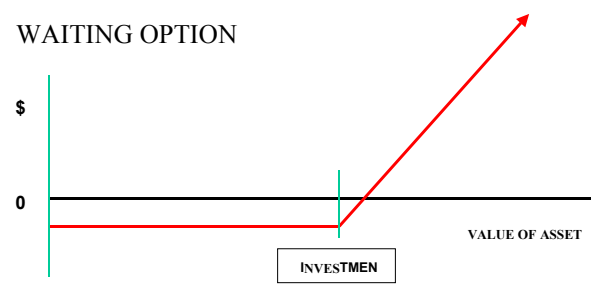
1. Introduction

The literature of real options starts with the papers of Myers (1977), Mac Donald and Siegel (1984, 1985,1986), Dixit and Pyndick (1995). This literature explores the analogy between financial options (calls and puts) and the flexibility every individual or investor has in the real world to respond to changing conditions of the business environment, taking advantage of the favorable states of the nature, and avoiding the losses of unfavorable states.
The basic real options described in the literature are the waiting option and the growth option (both similar to a "call" or a right to purchase) and the abandonment option (similar to a "put" or a right to sell). In this work we shall make use of the waiting and growth options associated to tender offers and acquisitions processes.

1.1 Waiting option

The waiting option reflects the potential flexibility to wait before taking a decision, getting access to new and better information and be in a better position to decide when circumstances make worth do it. This means that in a very uncertain environment the action of sinking investment all at once will be like making a bet. If the investor can wait and access to better information, she would always be in better position to improve the bet and avoid bad states of the nature, while taking advantage of good ones (the options to invest is kept "alive" because is more valuable than exercised).
In the following graph we can observe the analogy of the payoff of the waiting option with a call option.

Fig.1



MAX [V - I, 0]

V = VALUE of ASSET

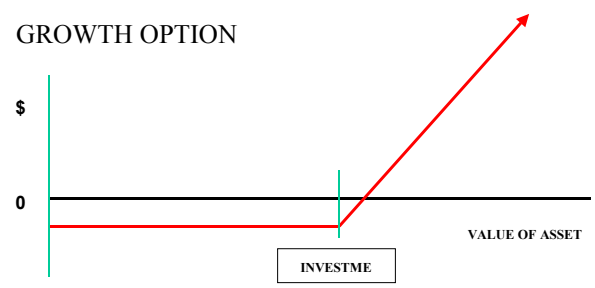
I = INVESTMENT

The call gives its holder the right to purchase an underlying asset of current value V by paying the exercise price I previously agreed. In the real options world, to make an investment is similar to exercise a call option, where the underlying asset is the current value of the future cashflows to be captured, and the cost of acquiring these cashflows is the amount I to be invested (exercise price). When deciding to invest or not, the investor is also deciding whether to kill or not her waiting option, so everytime someone invests is killing a waiting option. If the investor could wait to see how the random variables associated with the future payoffs turn on, would always be better off by keeping the option alive. However, it might be the case that keeping the option alive (not investing) carries on potential costs given by, for example, the entrance of another competitor seeking for the same cashflows, that in turn acts to reduce the value of the underlying asset (the cashflows now have to be divided between more participants). In this case the decision of investing may be the best way of locking the opportunity up, as a consequence of the trade off between the benefits of waiting and the costs associated to it.

1.2 Growth Option

Another real option depicted in the literature is the growth option (or the right to make follow up investments when states of the nature are favorable to the investor). In this real option, the investor reacts to good states of the nature by scaling up investments. Follow up investment gives the investor the possibility of capitalizing on the good states of the nature.
In the same tense as with the waiting option, the analogy with financial options comes associated to the call, which can be better appreciated in the following figure.

Fig.2



MAX [V - I, 0]

V = VALUE of ASSET

I = INVESTMENT

In this cases, the investor or entrepreneurs exercises the right to buy new cashflows associated to scaling up former investment (which is similar to paying an exercise price), which lets her capture more value when things turn out favorable, and avoiding investing further when states of the nature are unfavorable to the project. It can be easily seen that follow up investments are contingent on good states of the nature, and the investor has the right but not the obligation to invest (which in turn means she would not invest when nature does not show well for the decision).

Now we will see how these options are related to a learning process. If the investor or entrepreneur[1] could wait before undertaking an investment (e.g. there are no significant costs from waiting), she would "learn" about the true nature of the underlying asset and its associated cashflows.

1.3 Real Options and the Learning Process

There is a considerable amount of literature about the effects of the learning process. Arrow (1962) has one of the former works about the economic implications of "learning by doing". In Jovanovic and Mac Donald (1994a) firms improve their experience through innovation. There are also numerous works that explore the learning process through experimentation (Grossman et. al. 1977, Rob 1991, étc.). More recently, Bernardo and Chowdry (2002) explore the learning process by the firm about its own skills using a real options approach. It is our objective to apply the methodology of some sort of learning process in the context of real options to the process of gaining corporate control of a firm (through a takeover or acquisition).

The corporate control allows the main shareholder to define the orientation of the business giving her some discretion over business decisions (use of assets and other perquisites) in the context of respecting the rights of minority shareholders. An entrepreneur or investor may seek corporate control of a firm when it has a vision that the assets will be of much more value under her management. “Her management” conveys some sort of power over decisions like the use and disposal of assets, the use of resources, the types of products and services provided by the firm, etc. As we have mentioned before, a real option will always be associated to assets and resources that provide flexibility to react to the changing nature of business[2].

The entrepreneur or investor seeking control of a firm has an idea (not complete) about the cashflows generated by this firm (has "imperfect" information), and wants to learn more about the

[1] In the remaining I shall use the terms investor or entrepreneur interchangeable.
[2] An analysis of property rights and real options, can be found in Dapena 2002

true nature of these cashflows[3]. She also has an estimate of the value that can be created based on the current state of business and the improvements she can make; this is in many cases the foundation of the premium she is willing to pay above the market price. However, there will generally be situations where the investor could benefit from learning more about the firm, having a more accurate picture of its true nature and hence of the value to be created. Therefore by waiting or investing sequentially the investor can get access to more information about the firm before committing a large investment on a controlling stake. Thus, the investor can invest in a stock of shares that gives her a seat in the Board of Directors, which brings access to a different kind of information. Then she can decide whether is worth to make a large investment and gain corporate control of the firm, now with more information backing the decision and hence reducing the risk of an error of judgement.

2. The Value of Corporate Control

The possibility of deciding about the destiny of a firm, having power to transform opinions into courses of actions, and controlling and getting access to the flow of private information, has a value widely recognized in the market. This value is commonly known as the control premium and it is equal to the difference in value of a share belonging to a majority shareholder with respect to the value of a share of a minority shareholder.

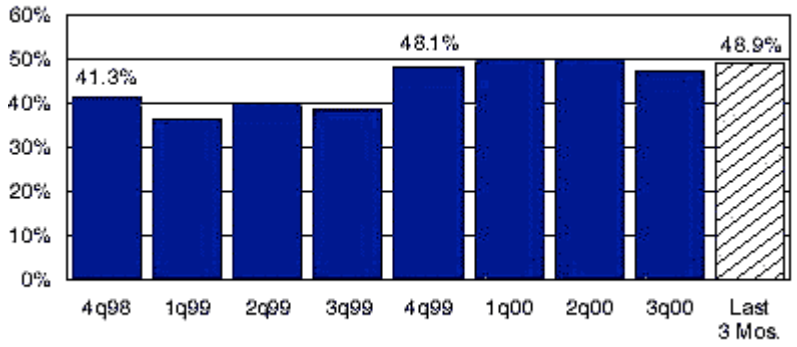
The corporate control comprises the possibility of taking decisions about:

1. Election of the management;
2. Compensation schemes;
3. Business policies and practices;
4. Acquisitions or disposal of assets;
5. Selection of business partners;
6. Payment of dividends;
7. Financing policy;
8. Etc.

3 If the investor had perfect information about the future cashflows, there would be nothing left to learn, which makes the waiting option completely valueless.

Pratt, Reilly and Schweihs (1995) quote premiums between 30 and 40% in transactions in the United States in the '80 and '90s. More recently, the premiums have oscillated among these figures, as we can see in the following graph.

Graph 1



Source: Mergerstat

A similar pattern is observed in transactions in Europe, with premiums reaching in some cases 80% of the former price of the share.

Among the reasons cited to justify the size of the premium, we can mention the existence of synergies between business, the access to valuable information and hence new business, and even the possibility of changing the direction of business in favor of the acquirer.

These reasons may give rise to opportunities of doing new business (“calls”), and opportunities of selling current business once in control (“puts”).

Even though there is no formal market of real options, the market where these "options" are trade is the market for corporate control, which has been object of regulation to prevent fraud and damages to minority shareholders.

2.1 The Tender Offer in Comparative Law

The comparative law brings in different systems, which ensure a fair treatment to minority shareholders when an investor or entrepreneur wants to take control of a company

Among the systems devised to protect minority shareholders, we can identify two, with a third one lying between in,

a. Obligatory Tender Offer.

b. Voluntary Tender Offer.
c. Partially Obligatory Tender Offer.

The first one stems from the English law, later adopted in France, Belgium and Italy. It states that everyone who wants to acquire a certain percentage of shares (which varies in different countries but generally over a 30%), must make a tender offer for ALL the outstanding shares.
This system has been originated on the principle of equity for all shares, giving the possibility to all the shareholders t liquidate their shares in case they want.
The voluntary tender offer is related with the American law. In the United States, the tender offer is not the only method to gain control, given that control can be gained as well by direct purchases of shares, or open market purchases. The protection to the minority shareholders comes through the application of severe regulations of disclosure of information about the transaction itself, and disclosure of the plans of the acquirer
In Spain there is an intermediate level, the Partially Obligatory Tender Offer, in which someone seeking control of a firm can make a tender offer for a certain percentage of shares (generally less than 100%), committing to purchase them at a specified price.

3. Analysis of the real options involved in the process of takeover r acquisition or corporate control

So far we have developed the most relevant details about the origins of value stemming from corporate control in terms of the decision associated to such a control.
Now we shall develop a simple model, which lets us identify the real options involved in a process of acquisition, the relation between them and their relation to the control premium.

3.1 A Time Discrete Model with Discrete States of the Nature

Consider a neutral risk investor or entrepreneur assessing whether to invest $I_c = I_1 + I_2$ in the corporate control of a firm[4]. At t=0 she has an estimation of the possible value of the cashflows and their probabilities associated, and based on this makes an estimation of the value V_i of the firm.

[4] Defining control as the percentage of total capital needed according to different legal systems to undertake actions like those described in section 2.

At t=1 there are two possible states of the nature, where the firm produces the cashflow F_h with probability p and F_l with probability q = 1-p (where $F_h>0$ and $F_l<0$). The entrepreneur can invest the full amount I_c at the beginning and gain access to control, or eventually invest a lower amount I_1 and decide later whether complete acquisition by investing $I_2 = I_c - I_1$ contingent on the state of the nature.

In short, at t=0 the investor or entrepreneur must decide between:

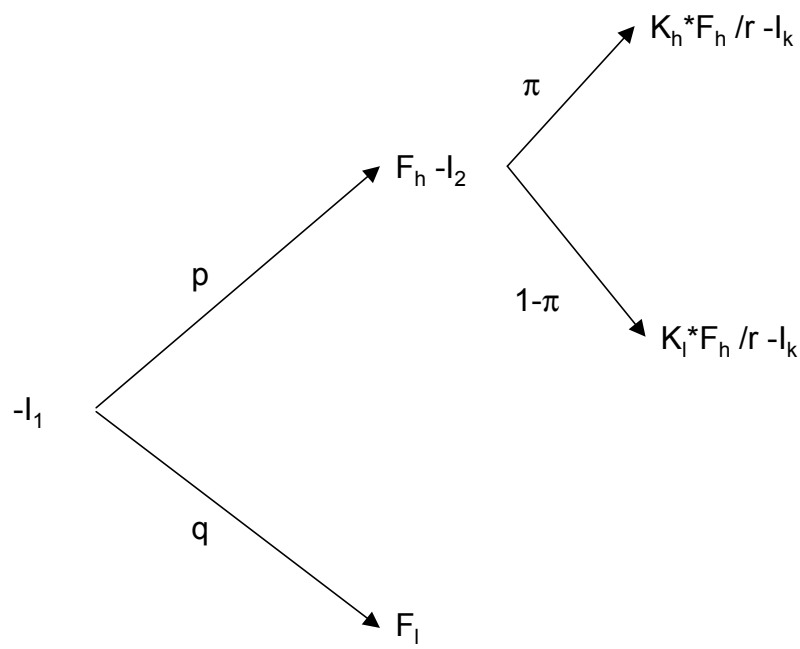
(i) invest $I_1 = I_c$ and gain control with an associated value equal to the discounted value of the expected cashflows.

(ii) invest $I_1 < I_c$, wait to see how nature unfolds and act in turn;

At t=1 the true state of the nature is revealed with its associated cashflow F_i. This revelation can be of two different sorts, public or private (which shall modify the results in terms of optimal decisions). Facing this revelation, the entrepreneur has the opportunity of scaling up the investment by a variable K_i that reflects the private added value of the control of the firm. This variable K is at first imperfectly known by the entrepreneur (she does not perfectly know how skillful she is in managing the firm's assets), and can adopt two possible values, K_h with probability π and K_l with probability 1- π. This multiply of value stemming from control is achieved by investing I_k.
We impose the restriction that all investments are sunk cost once made, and all that matters is the realizations of F.
By simple inspection we can see that if the revealed cashflow is F_l, the investor does not choose to invest in gaining control of the firm, and hence does not make follow up investment, because should this happen, she would be multiplying her losses (measured by the variable K multiplying the cashflow F_l <0). If this was the case, the optimal decision is to write off the loss I_1, and abandon the project. On the contrary, if the realized cashflow is F_h, then the investor must decide whether to invest and get full control of the firm, and hence capture the growth option embedded.

The sequence of events can be depicted in the following figure,

Fig. 3



where the net payoff associated to the project is:

$$\text{Expected Net Present Value} = - I_1 + \{F_i + \text{Max}\,[F_i - I_2 + \text{Max}\,(E(K_i)* F_i/r - I_k, 0), 0]\}/(1+r) \quad [1]$$

The maximum function allows us to eliminate all the negative payoffs, reflecting the flexibility associated to the real options.
At t=2 every uncertainty is resolved, or there are no important decision to be taken with respect to the original decision of gaining or not control, and the value of K is realized showing either a profit or a loss

3.2 Solution

To find a solution we should state what would the behavior of the variable I_2 (follow up investment needed to gain control) be.

It is clear that at t=0, the value of seizing control of the firm has to be consistent with its market value (the extra value for the investor originated by scaling up investments and cashflows by the variable K_I is private and not realized yet for anyone), where[5]:

$$V = I = E[F]/r = [p * F_h + q * F_l]/r \qquad [2]$$

This means that if the investor wants to gain control at t=0, she should pay the current market value of the controlling stake, $I_c = I_1 = V$[6]. On the other hand, a lower participation (a no controlling stake) would imply spending or investing less, $I_1 < I_c$.

At t=1 there are two interesting situations related with the investment I_2 needed to complete the acquisition of the controlling stake. The reaction or level of this investment shall modify substantially our results provided the assumptions made about the dynamics I_2.

We shall treat two extreme cases and which are related with the reaction of I_2 to the revelation of the cashflow F at t=1. In the first case F is privately observed by the investor, hence the amount I_2 to be invest at t=1 remains the same as what was expected to be paid at t=0 ($I_2 = I_c - I_1$). In the second case, the realization of F is publicly observed, so the value of I_2 is adjusted to reflect what the market knows about F_i and hence makes more costly (or less if the realization is a bad state of the nature) to gain control of the firm. For the sake of the analysis, we shall normalize I_c= 100%

Case 1. Value $I_2 = I_c - I_1$

The problem to be solved is the maximization of [1] with respect to the control variable I_1 subject to the following restrictions:

$$I_2 = I_c - I_1 \qquad [3]$$
$$I_1 \geq 0$$
$$I_2 \geq 0$$

Solving the problem we obtain a corner solution where $I_1 = 0$, $I_2 = 100\%$, which means that under the conditions mentioned, the investor invests nothing in the first period, and waits for the realization of the events in order to decide at t=1 whether is worth gaining control of the firm or not. This result is intuitively correct, given that by following this strategy the investor avoids to invest a dollar in the first stage and suffer a loss in the second stage if the state of the nature is unfavorable ($F_l < 0$). The investor holds a waiting option, whose value stems from avoiding having invested when the true state of the nature becomes the bad one, being this option value

[5] Assuming that once one of the two possible cashflows is revealed, remains the same forever.

maximized by following the mentioned strategy. Every dollar the investor invests at t=0 in I_1 has an expected loss associated, with no real benefits in exchange (intuitively, the investor, when facing the decision to invest at t=0 or at t=1, and with no cost associated, will always prefer to invest at t=1 with more information in hand), and with no change in the underlying value of the opportunity. The present value of the expected loss avoided by the investor is given by:

$$q^* I_l/1+r \qquad [4]$$

given that every dollar of investment is lost if the revealed state of the nature is L. The maximum value of the waiting option will be given by the difference between the net expected value with investment of I_l= 0% (wait and maximize the value of the option) and I_l= 100% (invest all at once at t=1).

With I_l= 0%

$$E[F]/(1+r) - I = p^* \{F_h - I_c + [E(K)^* F_h/r - I_k]\}/(1+r) \qquad [5]$$

and with I_l= 100%,

$$E[F]/(1+r) - I = \{p^* [F_h + [E(K)^* F_h/r - I_k]] + q [F_l]\}/(1+r) - I_c \qquad [6]$$

where [6] minus [5] gives us the maximum value of the waiting option:

$$\text{Waiting option} = q^* (I_c - F_l)/(1+r) \qquad [7]$$

knowing that $F_l < 0$. The value of [7] provides the investor the maximum value attainable by waiting to see the development of the events. In case the state of the nature is unfavorable, the investor avoids having invested the full value I_l=I_c which is consistent with [4] (the loss F_l does not affect the position).

Case 2. Value I_2= $(F_h/r)^*(I_c - I_l)/I_c$

This situation is intended to show the other extreme, where the value of the company (and hence the controlling stake) fully reacts to the disclosure of the state of the nature. In this case the

[6] For simplification purposes we obviate the fact that pushing the market with such a purchase will surely rise the market price of the remaining shares. This simplification may sound strong, but including it in the analysis does not change results.

problem of the investor is to maximize [1] with respect to the control variable I_1 subject to the same restrictions and adding a new one given by:

$$I_2 = (F_h/r)*(I_c - I_1)/I_c \qquad [8]$$

The solution now of this problem is substantially different. In this case the investor must make a trade off between the value associated to wait (avoiding a loss) associated to the control variable I_1 according to [4] :

$$q* I_l/1+r$$

where each increment of I_1 translate in an expected cost of :

$$q*\Delta I_1 /1+r \qquad [9]$$

and the extra value added coming from the savings in the exercise price of the growth option I_2 according to [8]:

$$p*(F_h/r)*(1/I_c)*\Delta I_1 /1+r \qquad [10]$$

In the optimum, the amount to invest I_1 is up to a point where the marginal cost of investing at t=0 [7] (killing a waiting option) becomes equal to the marginal benefit [8] of saving on the exercise price, with the following expression (where p= 1-q):

$$q / (1-q) = F_h/(r*I_c) \qquad [11]$$

The right hand side term in [11] is in general greater than the left hand side term, given that according to the market value at t=0 of I_c

$$I_c = E[F] = [(1-q)* F_h /r+ q * F_l/r] / 1+r$$

for high values of q, in equilibrium and to avoid $I_c<0$ (given that $F_l<0$) we need high value of F_h and then:

$$q / (1-q) < F_h/(r*I_c) \qquad [12]$$

Then, the maximum is obtained with I_1 = 100%, I_2=0, which means that the best strategy for the investor seeking control is investing all at once at t=0, taking into consideration an any kind of expected loss arising from bad states of the nature and killing the waiting option. The additional restriction imposed implies that the market value of the remaining shares needed to gain control is adjusted based on the revelation of the state of the nature, which in turns increases (in the favorable state) the exercise price of the growth option and hence reduces its value. The investor now cannot buy the remaining share needed for control at the former price $I_2 = I_c - I_1$, because agents have adjusted their price based in the new information in the market. The private information held by the investor is related only to the growth option (the own skills associated to the management) but the information related to F_i is public. In general, the value of the growth option is given by:

$$\text{Growth option} = \text{Max}\ (V - I_2, 0) \qquad [13]$$

where I_2 is the investment needed to acquire the underlying asset of value V, whose value comes from the expected value of K multiplied by the realized cashflow F_i, net of new investments I_k. If the investor does not gain control at t=0, the exercise price of the growth option is increased in the good state of the nature, making more costly for her to gain control and seize this option, and hence reducing the value of the growth option.

3.3 Discussion

The results obtained are based on extreme cases. In both situations the information of the investor associated to the growth option is private and unknown to the market (she knows the distribution of the variable K_i which being a random variable can even give rise to the situation where the realized benefits are not as bigger as expected at first), but the main difference between these two cases is the reaction in the level of the variable I_2 needed to gain control.

In the first case the revelation of the state of the nature and hence of F_i is private and seen only by the investor, so the market does not react by increasing the price of the remaining shares of the controlling stake. We could think that the investor sees a private signal which is highly or perfectly correlated with F_i, which she was searching for and which lets her better infer the true nature of her growth option. This signal, which could be based on better interpretation of private information of the firm, is observed only by the investor, and allows her to handle a better estimation of her growth option. It is because of this that the investor, in the optimum, does not

invest any amount given that the signal is revealed to her anyway and for free, and providing this strategy an expected value given by [3].

In the second case, the signal is of public access in the sense that is not only observable to the investor, but also to the market as a whole, which in turn adjusts the estimation of price. As a consequence, it becomes much more expensive for the investor to wait until the realization, therefore (and given that the growth option is private) the investor prefers to gain control at t=0 instead of building control by sequential investments while observing the realization of events. This action in turn implies killing partially or totally the waiting option and its associated value, but saving the increase in the price of the controlling stake I_2 if the state of the nature is favorable and F_h is realized, keeping intact the value of the growth option. It is verified that the investor shall prefer to kill the waiting option fearing that the public knowledge of the true state of the nature will increase the cost I_2 of gaining control of the firm at t=1. This fear and its related cost overcome losing the investment if the state of the nature is unfavorable (recall that ex post the amount invested is a sunk cost). What outweighs this loss is the high cost of acquiring control in the second stage if the state of the nature is favorable.

Useless to say that none of these extremes cases is generally verified in reality; we always find an intermediate point between these two extremes. Our model suggests that any internal solution (I_1>0, I_2>0) will depend on the degree of reaction of I_2 to the realization of F, where in our model this reaction is given by the private or public nature of access to F. It is also true that the investor has to construct necessarily a position I_1 >0 (eliminating the extreme I_1=0) to be in a position to get access to any kind of sensitive information about the firm and its cashflows. The expected sequence of a takeover or acquisition process may start by acquiring a minority stake ($I_c > I_1 > 0$) which lets the investor "buy" information[7] about the true nature of the firm (even in a reduced scale or imperfectly), and then a decision about acquiring control or not of the firm. As we could observe in the sequence showed in figure 3, even in the case that information is privately acquired and the state of the nature is favorable, the firm must also invest after an amount I_k, with an uncertain K_i showing its true nature, which means that it could be completely rational to take control of a firm to find next that the ability to exploit the growth option is not the one thought before (being the realized ability K lower or higher than expected, and hence the profits).

The extremes developed as cases 1 and 2 let us infer that the decision of gaining control of a firm at a first stage (the amount to be spent initially) will be a function of the ability of the investor to accurately estimate values. As long as this initial shareholding allows the investor to observe and better process the true nature of events, she will always prefer to wait; as long as this ability is public and shared with others becoming public (hence reflected in prices) the investor will gain control in the first stage, killing the waiting option. This is perfectly rational, since when the

[7] "Buy" in the sense of acquiring a real option as in Dapena 2002.

information becomes public, the exercise price of the controlling stake needed to access the growth option (I_2) will start to rise, reducing the value of such an option. In a sense it acts as a dividends distribution in financial options, reducing the value of the call option on the stock; here it increases the exercise price, and therefore a reduction in the premium to be paid. Both a decrease on the current value of the underlying asset or an increase on the exercise price threaten the value of the growth option.

In general terms, every agent holds an option to buy a firm. Even more, every one of us holds a right to buy any asset. In the context of our model, it means that any investor derives value from estimations of growth for the company. If this was the case, an option shared by all people is worth nothing, in the sense of Dapena (2002). Therefore, the "property" of the growth option embedded in the project will be stronger for those who build up minority shareholdings in the targeted firm, which allow them to count with better information about the true nature of the firm.

3.4 A Numerical Simulation

Now we shall proceed with a numerical simulation intended to show the results of the maximization problem in the simplest way given the value of some parameters. In the following numerical simulation we can observe the value of the real options associated with a likely corporate control of a firm.

We set values for the following parameters

Table 2

F_h	14.0
F_l	-6.0
p	55.0%
q	45.0%
K_h	1.5
K_l	0.9
π	50.0%
$1 - \pi$	50.0%
I_k	100.0
r	5.0%

which implies a value of the investment I_c equal to 100 at t=0, with values for F_h and F_l and their associated probabilities, a discount rate equal to r (risk free and the relevant in the economy given our investor is risk neutral) and the multiplier variable K_i which can adopt one of two values with

probabilities π and 1- π. Now with these figures, we obtain the value for the following endogenous variables:

Table 3

I_c	100
K_h*F_h/r	420
K_l*F_h/r	252
$K_h*F_h/r - I_k$	320
$K_l*F_h/r-I_k$	152
F_h/r	280

With these values, we can now run the optimization problem for each of the two cases analyzed. In **case 1**, where the realization of the state of the nature and its associated cashflow is privately observed by the investor and the new information is not fully reflected in the price of the share, the maximum value for the net present value of the investment in control al t=0 is:

$$VPN = 79.8$$

In the following table we can observe how the net and brute present values change when the initial investment I_1 changes

Table 4

I_1	I_2	E[F]	E[F] - I_1
0%	**100%**	**79.8**	**79.8**
5%	95%	82.6	77.6
10%	90%	85.3	75.3
15%	85%	88.1	73.1
20%	80%	90.8	70.8
25%	75%	93.6	68.6
30%	70%	96.3	66.3
40%	60%	101.8	61.8
50%	50%	107.3	57.3
60%	40%	112.8	52.8
70%	30%	118.3	48.3

Case 2 has the difference, the exercise price of the investment needed to gain access to control of the firm I_2 responds to the relevant state of the nature. In this case, both the investor and the public as a whole have the ability to observe at a second stage the relevant state of the nature, and

the price adjusts to reflect this new information. We mentioned that if this was the case, in the favorable states of the nature the price of the share rises which makes more costly to gain control of the company, increasing the exercise price I_2 of the growth option and therefore reducing its value.

The result is consistent with our previous findings, where I_1= 100% with a net present value of:

VPN= 34.8

The investor forecasts the increase in the cost of the exercise price, which makes her increase her investment in the first stage to avoid this cost. So in the extreme it becomes optimal for the investor gain control and hence the growth option at t=0 by making I_1= 100%, to avoid the costs associated with the increase in the exercise price I_2 should the favorable state of the nature become true. With this strategy, the investor "kills" the waiting option whose value arises from avoiding investments should the unfavorable state of the nature become true, but this costs is more than compensated by the savings in I_2 and the fully ownership of the growth option at the original exercise price.

The following table shows us different combinations of I_1 and I_2 and the net present value associated:

Table 5

I_1	I_2	E[F]	E[F] - I_1
100%	**0%**	**134.8**	**34.8**
95%	5%	127.1	32.1
90%	10%	119.4	29.4
85%	15%	111.7	26.7
80%	20%	104.0	24.0
75%	25%	96.3	21.3
70%	30%	88.6	18.6
60%	40%	73.2	13.2
50%	50%	57.8	7.8
40%	60%	42.4	2.4
30%	70%	27.0	-3.0

where we can observe how the net present value changes in response to different combinations of amounts I_i invested.

4. Conclusions

As we have developed in this paper, the value of the premium of control which is usually associated in a direct way to the extra value an investor is willing to pay, can be seen as a real option whose underlying asset is expected discount value of the cashflows associated to the set of assets and resources belonging to the firm. This value is sensitive to the exercise price needed to get access to the control of the firm.

Control over the assets and resources of a firm give the majority shareholder some sort of discretionality on their best use. Corporate control has added value for an investor since it gives her degrees of freedom about the use of assets, sources of finance, salaries, etc.

In most of the cases control arises from different categories of tender offer as we have seen in section 2.

The acquisition process involves real options where the investor initially holds a waiting option associated to the whole investment. This waiting option is common to all investor in the sense that every one can invest at any time and at the right price. By purchasing minority shareholdings, the investor tries to appropriate potential growth options. These growth options are private opportunities for the investor, and their realizations are contingent on the true nature of the cashflows in each state of the nature. There are some sort of minority shareholdings that allow the investor to get access to better information and be in a better position to assess the real value of the growth options associated to the assets of the firm, by doing some learning and taking full advantage of the waiting option.

On the other hand, the control of the firm (and hence the growth option) has an exercise price which may change according to the revelation of the state of the nature. A very important point regarding the process off gaining control of a firm is given by the reaction of this exercise price to the revelation of the state of the nature. As long as the exercise price does not react to the release of information (the events and signals are privately observed), the investor is "almost" proprietary of the option, because observes the events and decides whether to invest or not. This position lets her avoid wasting money by investing and then having a bad state of the nature revealed. If, on the other hand, the prices (or public perception of value of the firm) reacts to the state of the nature revealed, the value of the remaining investment I_2 or exercise price needed to gain full control over the assets of the firm in the good state of the nature will increase, hence lowering the value of the associated growth option (in a similar fashion to an unexpected distribution of dividends in options like calls). Foreseeing this situation, the investor decides to save money and get full control over the assets of the firm and hence over the growth option at t=0. This decision "kills" the value of the waiting option, but the benefits may outweigh the costs associated to this loss of value.

The investment at the first stage I_1 can also adopt different means, in the sense that getting a participation in the capital of the firm it is not the only way to accomplish this. If, for example the firm is private and there is no easy way to purchase minority shareholdings before making a tender offer, the investor can invest in a similar but smaller firm to learn more about the market and the true nature of the business environment, giving rise to better estimations of cashflows and values associated

As we have mentioned, in most of the cases we shall find some sort of intermediate case (given by $I_1>0$, $I_2>0$) between the two extremes developed and their corner solutions, where the exercise price of the controlling stake and hence the growth option reacts to some extent but not in the way described (so we can find some sort of mixed between private observation of events and public observation); there will also be some legal aspects related to minimum or maximum participation required to make a tender offer, and we shall also find a wider distribution of skills and abilities to better acquire and process information and signals.

An extension of our work would be to collect information from the market about the sequence of decisions about investments followed by investor when acquiring control of firms, the size of the premium paid as an approximate to the value of the real options and the motives to exercise the option at a particular time. It can also be observed the reaction of the value of the investment needed to get control of the firm, and how it accelerates or not the decision of investing.

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